3D FINITE FRACTURE MECHANICS UNDER MODE I LOADING: THE FLAT ELLIPTICAL CRACK

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Abstract
In recent years, the Finite Fracture Mechanics approach, originally proposed by Leguillon in 2002, has been applied successfully to several material and geometrical configurations. However, up to now, most of the applications were restricted to two-dimensional geometries. In the present paper, we provide an insight to a simple yet challenging three-dimensional case, namely the flat elliptical crack. Results are provided in analytical form.

1. Introduction

Finite Fracture Mechanics (FFM) relies on the assumption that cracks, at least at onset, grow in a finite, discrete way, i.e. by crack steps [1]. FFM has proven to be an effective fracture criterion for predicting applied stresses causing/producing/originating the crack onset in 2D geometries. Some attempts have been done to extend the criterion to 3D geometries [2,3]. The main difficulty lies in the fact that the failure stress depends not only on the size of the crack step, but also on its shape. Under mode I loading conditions, the dependence on the shape has no effect in 2D geometries while it has in 3D.

2. Results

In the present paper, we focus the analysis to flat elliptical cracks under uniform remote stresses normal to the crack plane (Fig.1a). Note that the problem can be seen as a straightforward generalization of the penny shaped crack geometry recently faced by FFM in [4], but here the radial symmetry does not hold any more.

Onto the crack plane (x,y) only the normal stress component σ_z is acting. Its value over the whole crack plane can be derived from the complete solution by Green & Sneddon [5]. It reads as:

$$\frac{\sigma_z}{\sigma_\infty} = 1 + \frac{1}{E(k^2)}\left[\frac{a}{\sqrt{\xi}}\sqrt{\frac{b^2+\xi}{a^2+\xi}} - E\left(\arcsin\frac{a}{\sqrt{a^2+\xi}} \,\Big|\, k^2\right)\right] \quad (1)$$

where a and b are respectively the major and minor semi-axes of the ellipse describing the crack, k is the eccentricity, σ_∞ is the remote applied stress, $E(\cdot|\cdot)$ is the incomplete elliptical integral of the second kind and $E(\cdot)$ is its complete counterpart; ξ is an ellipsoidal coordinate whose relationship with the cartesian coordinates (x,y) on the $z = 0$ plane is:

$$\frac{x^2}{a^2+\xi} + \frac{y^2}{b^2+\xi} = 1 \quad (2)$$

Because of the applied remote stress, the elliptical crack opens, taking the shape of an ellipsoid whose semiaxes are a,b and $w_{\max}$. The third one, representing half of the crack opening at the crack centre, is:

$$w_{\max} = \frac{2\sigma_\infty b}{E(k^2)E'} \quad (3)$$

E' being the material Young's modulus in plane strain conditions. Based on Eqs. (1), (2) and (3), we can apply LEFM assuming an infinitesimal crack extension of (any) elliptical shape. A reasonable assumption is to assume an *iso-stress* crack increment. It can be shown that such an assumption is tantamount assuming a crack increment orthogonal to the crack front proportional to the square of the SIF acting at that point or, equivalently, assuming that the *equivalent* SERR (i.e. the SERR value to be compared with the fracture energy to have crack growth) is the *contraharmonic* mean of the SERR values along the whole crack front. However, an infinitesimal crack growth where only the *minor axis* is growing always provides the maximum energy release and, thus, it is preferential from the LEFM point of view.

Analogous analysis can be performed by means of the FFM approach. In this case, however, results are different: it always exist a *threshold* crack size below which the iso-stress (now finite) crack extension is preferential since, coupled with the stress requirement for crack onset, it occurs at a lower remote stress value. This threshold value depends on the ellipse eccentricity and on Irwin's length, so that we can conclude that minor-axis crack growth will take place for high eccentricities, large crack sizes and highly brittle materials, while iso-stress crack growth will prevail for low eccentricities, small crack sizes and not so brittle materials, see Fig. 1b.

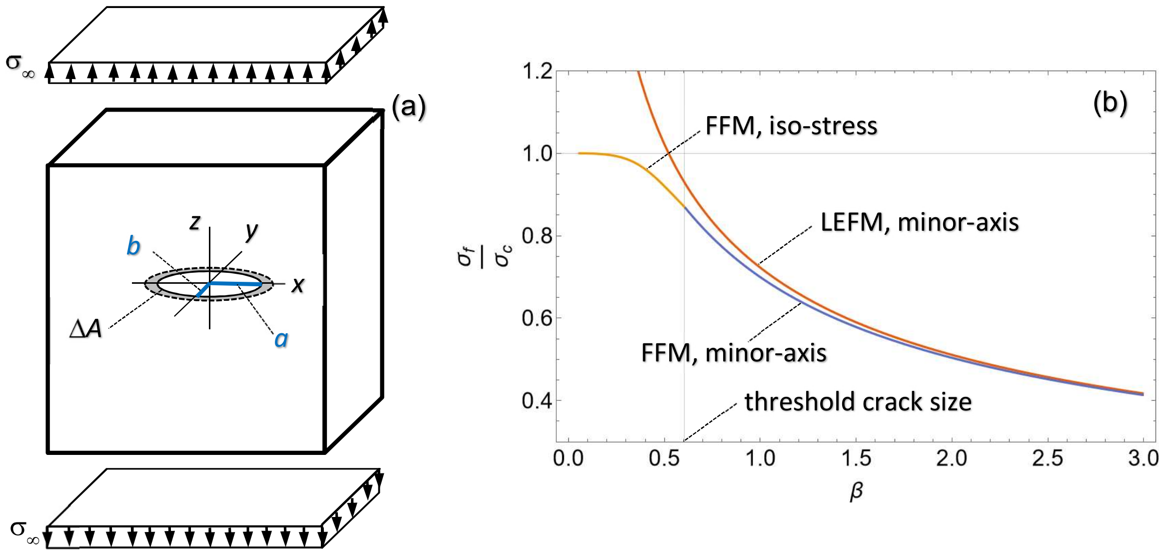

Fig.1 – Flat elliptical crack (a) and strength size effect (b) for $b/a = 0.5$; ΔA is the finite crack extension, σ_f is the failure remote stress, σ_c is the material tensile strength, $\beta = b / l_{ch}$ the dimensionless crack size.

3. Conclusions

The flat elliptical crack under mode I loading conditions have been faced by LEFM and FFM. FFM always provides lower values, so that uncritical application of LEFM can lead to unsafe design.

References

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