## USING MIXED FINITE ELEMENTS AND REMESHING TO ASSESS BRITTLE FAILURE USING THE BEREMIN MODEL

Jacques Besson<sup>1\*</sup>, Amar El Ouazani Tuhami<sup>2</sup>, Nicolay Osipov<sup>2</sup>

<sup>1</sup> MinesParis, PSL, Centre des Matériaux, UMR CNRS 7633, France, <sup>2</sup> Transvalor, S.A., France, jacques.besson@minesparis.psl.eu

**Abstract:** A computational strategy to evaluate the Weibull stress for the Beremin model is proposed to simultaneously solve problems caused by volumic locking and extreme element distortion at the crack tip. It is based on the use of mixed elements and remeshing. It is shown that a single simulation can be used to evaluate the Weibull stress for any range for the CTOD at failure.

## 1. Introduction

The probability of brittle failure in ferritic steels can be modeled using the well-established Beremin model [1]. This model requires the evaluation of a Weibull stress  $\sigma_w$  as:

$$\sigma_w = \left(\int_{\Omega} \max_{\text{history}} \left(\sigma_I H(\dot{p} > 0)\right)^m \frac{d\Omega}{V_0}\right)^{1/n}$$

where  $\sigma_I$  is the maximum principal stress, p is the accumulated plastic strain and H is the Heavyside function. The condition  $\dot{p} > 0$  expressed the fact that failure only occurs if plasticity is active. m is the Weibull exponant and  $V_0$  is a reference volume. Despite the apparent simplicity of the model, the evaluation of the failure probability faces some challenges when dealing with cracks. The mesh size must be sufficiently small to properly discretize the failure process zone ahead of the crack tip. A finite strain formalism must be used to properly capture the Crack Tip Opening Displacement (CTOD) corresponding to crack blunting and the stress distribution ahead to the crack tip. However, the mesh can become too distorted so convergence cannot be achieved. In addition, volumetric locking may also occur due to the quasi-incompressible behavior of the metal. This leads to spurious high stresses which lead to a poor evaluation of  $\sigma_w$ . This problem is particularly critical as the Weibull exponent m usually takes high values between 10 and 30. The purpose of this work is to propose a computational methodology to better evaluate the Weibull stress. It uses a mixed finite element formulation to better control pressure fluctuations due to quasi-incompressibility together with remeshing to keep an optimal element size and shape around the crack tip. All simulations were carried out using the ZSET software [2].

## 2. Results

**Mixed elements:** A formulation based on a mixed-enhanced treatment involving displacement, pressure and volume variation is used to avoid volumic locking [3, 4]. Triangular elements allowing for remeshing are used with quadratic interpolation of displacements and linear interpolation for pressure and volume variation. Fig. 1 evidences the strong fluctuations of  $\sigma_I$  when using a standard displacement-based formulation (although reduced integration is used). Fluctuations disappear using mixed elements. Note that linear square (3D) or hexahedra (3D) using the B-bar method can also be used to eliminate locking [5]. However, they cannot be used together with remeshing.

**Remeshing:** Using remeshing allows the optimization of mesh design to keep an optimal element size and shape. Remeshing is performed based on the current value of the CTOD as schematically depicted in fig. 2. A circular zone centered ahead of the crack tip (distance:  $\beta$ CTOD) having a radius proportional to the CTOD ( $\alpha$ CTOD) is meshed using a fixed mesh size which is a fraction of the current CTOD (CTOD/k).



Figure 1: Principal stress ( $\sigma_I$ ) at crack tip for standard and mixed elements (undeformed configuration) for a CTOD equal to 0.4 mm. The box indicates the zone where volumic locking is observed.

The following values, which provide converged results, are used here:  $\beta = 2$ ,  $\alpha = 4$  and k = 4 were used. Remeshing is triggered each time the CTOD increases by a given factor (50% in the present case).



Figure 2: Mesh design close to the crack tip

Figure 3: Evolution of the Weibull stress as a function of CTOD.

**Evaluation of the Weibull stress:**  $\sigma_w$  is evaluated with m = 20 and is plotted as a function of the CTOD in fig. 3 for the standard elements and the mixed elements with or without remeshing. It is shown that  $\sigma_w$  diverges in the first case due to locking. Using mixed elements solves this problem but the simulation stops at some point due to excessive element deformation. Using remeshing solves both problems. Results are considered to be independent of the initial meshed radius as soon as the CTOD is larger than 10 times this radius.

## 3. Conclusion

The proposed methodology has a particular interest in the case of ductile to brittle transition for which the values of the CTOD at failure or ductile crack initiation are highly scattered. A single simulation with an initial crack tip radius as small as necessary can be used. The method is also available for 3D cases using tetrahedra.

- [1] F. M. Beremin, A local criterion for cleavage fracture of a nuclear pressure vessel steel, Met. Trans. 14A (1983) 2277–2287.
- [2] Zset, Non-linear material and structure analysis suite. http://zset-software.com, Tech. rep.
- [3] R. Taylor, A mixed-enhanced formulation for tetrahedral finite elements, Int. J. Numer. Meth. Engng 47 (2000) 205-227.
- [4] M. Bellet, Finite element analysis of compressible viscoplasticity using a three-field formulation. application to metal powder hot compaction, Comp. Meth. Appl. Mech. Engng 175 (1–2) (1999) 19–40.
- [5] A. Jivkov, D. Sarzosa Burgos, C. Ruggieri, J. Beswick, R. Savioli, P. James, A. Sherry, Use of local approaches to calculate changes in cleavage fracture toughness due to pre-straining and constraint effects, Theor. Appl. Fract. Mech. 104 (2019) 102380.