ON THE LIMIT EQUILIBRIUM OF SMALL-SCALE INTERFACIAL SHEAR CRACKS AT THE CORNER POINT OF THE INTERFACE OF A PIECEWISE HOMOGENEOUS COMPOSITE

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Abstract

A plane static symmetric problem of elasticity theory for a piecewise homogeneous isotropic plane with an interface in the form of sides of angle, containing small-scale interfacial shear cracks at a corner point and a loaded internal semi-infinite crack, is considered. The exact solution to this problem is constructed by the Wiener – Hopf method in combination with the apparatus of the Mellin integral transform. The stress intensity factor at the tips of interfacial cracks was determined and the nature of the change in the breaking load was studied.

1. Introduction

In problems for piecewise homogeneous composite materials containing cracks at the interface between media generally assumed that the interface between the media is smooth, primarily rectilinear. However, it is precisely near the corner points of a non-smooth interface between media that are stress concentrators that one should expect a discontinuity and the initiation of cracks emanating from them in the first place. In such a situation, it seems relevant to use an approach based on the decomposition of the problem under consideration into external and internal problems. This allows the transition from the problem of fracture mechanics of composite materials about interfacial cracks originating at the corner points of a piecewise homogeneous body to the elasticity problem for a wedge-shaped body with an interfacial crack at the vertex and a condition at infinity that takes into account the influence of an external field. Within the framework of this approach, using the Wiener – Hopf method, an exact solution to a plane static symmetric elasticity problem is constructed for a piecewise-homogeneous isotropic plane with an interface in the form of sides of angle, containing small-scale interfacial shear cracks and a loaded internal semi-infinite crack.

2. Problem statement

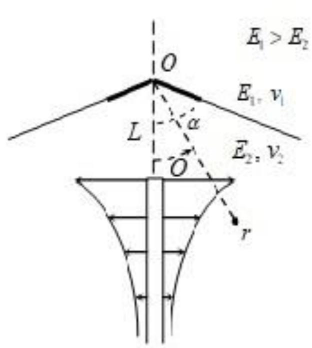

Fig.1

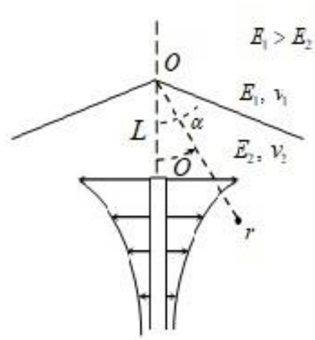

Fig.2

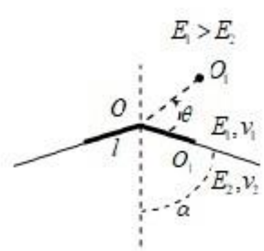

Fig.3

A plane static symmetric problem of elasticity theory for a piecewise-homogeneous isotropic plane with an interface in the form of angle sides, containing small-scale interfacial shear cracks originating at a corner point and a loaded internal semi-infinite crack is considered (Fig.1). It is assumed that the lengths of interfacial cracks are much less than the distance from the corner point to the tip of the internal crack. The faces of a semi-infinite crack are under the action of pressure, which is distributed according to the law F / r^2, $r \geq L$ (F is a given positive constant, which has the dimension of force). The task is to determine the stress intensity factor at the tips of interfacial cracks and to study the nature of the change in the breaking load.

The problem under consideration can be decomposed into external and internal problems. The external problem in relation to the problem as a whole is the problem of elasticity theory for a piecewise homogeneous isotropic plane with an internal loaded semi-infinite crack (Fig.2). The internal problem is a plane static symmetric problem of the theory of elasticity for a piecewise homogeneous isotropic plane with an interface in the form of sides of an angle, containing cracks of finite length, emerging from a corner point and located on this interface (Fig.3). Exact solutions of the corresponding elasticity problems are constructed by the Wiener – Hopf method.

3. Analysis of results

Using the solutions obtained, we determine the stress intensity factor at the tip of an interfacial crack in the problem of fracture mechanics of composite materials shown in Fig.1:

$$K_{II} = G(\alpha, e_0, v_1, v_2)\left(\frac{l}{L}\right)^{\lambda_0+1/2} \frac{F}{L^{3/2}}.$$

Here $G(\alpha, e_0, v_1, v_2)$ is known function, and λ_0 is the unique root on the interval $(-1;0)$ of some characteristic equation.

Equating the right side to the critical value of the stress intensity factor K_{IIc}, we obtain the following formula for determining the breaking load F_b:

$$F_b = \frac{K_{IIc} L^{\lambda_0+2}}{G(\alpha, e_0, v_1, v_2) l^{\lambda_0+1/2}}.$$

Dependencies of the dimensionless breaking load on the angle α for different values of the ratio of Young's modulus ratio $e_0 = E_1 / E_2 > 1$ are shown in Fig.4 ($l / L = 0{,}03$; $v_1 = v_2 = 0{,}3$). In the figure, curves 1 – 4 correspond to the values $e_0 = 2;\ 3;\ 5;\ 10$.

These data indicate that the smaller the distance between the corner point O and the tip of the internal crack, the lower the breaking load. The longer the length of the interfacial cracks, the lower the breaking load. With an increase in the angle α, the breaking load first decreases, then increases. Values e_0 equal to 2; 3; 5; 10 correspond to the values of the angle of a least breaking load equal to $40{,}3^\circ; 36{,}5^\circ; 34{,}1^\circ; 30{,}7^\circ$. The greater the Young's modulus ratio $e_0 = E_1 / E_2 > 1$, the lower the breaking load and the smaller the angle of the least breaking load.

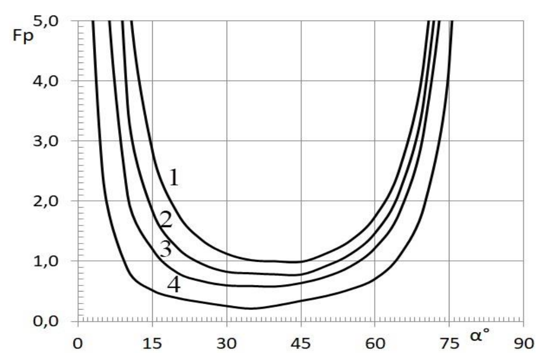

Fig.4