SINGULAR ELASTIC SOLUTIONS IN CORNERS AND CRACKS WITH SPRING BOUNDARY CONDITIONS WITH VARYING STIFFNESS

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Abstract
Singular elastic solutions in corners and cracks with spring boundary conditions with varying spring stiffness are studied. First, a novel analytic procedure is developed for the antiplane strain case. Then, some general observations obtained are checked for the plane strain case by using a FEM code. Finally, applications of these observations in a suitable computational implementation of the Coupled Criterion of Finite Fracture Mechanics are discussed.

1. Introduction
Linear elastic isotropic corners with spring boundary conditions with varying stiffness are considered. In particular, clamped-spring, i.e., homogeneous Dirichlet-Robin, boundary conditions at planar corner faces are considered. The corner material is assumed to be homogeneous. Spring boundary condition, given by a continuous distribution of independent linear elastic springs (Mantic et al, 2015), can be considered, e.g., as an approximation of a thin adhesive layer. The stiffness of these springs is considered to vary as a function of the distance to the corner apex. This variation can represent, e.g., the varying thickness of the adhesive layer, or the varying degree of damage in the layer. One of the objectives of this work is to deduce singular elastic solutions in the form of asymptotic series in the vicinity of the apex of such corners. These series are composed by the main terms, solutions of the corresponding clamped-free, i.e., Dirichlet–Neumann, corner problems, and the associated finite or infinite series of the so-called shadow terms determined by solving systems of recursive inhomogeneous Dirichlet–Neumann or Dirichlet–Dirichlet problems for the corner, cf. Jiménez-Alfaro (2020).

2. Results
A novel analytic procedure is developed for the deductions of the asymptotic series of singular solutions in corners with clamped-spring boundary conditions with varying spring stiffness under antiplane strain assumptions. This procedure is a generalization of the procedure, suitable for computational implementations, developed by Jimenez-Alfaro et al (2020) considering springs with a uniform stiffness along the corner face. The complex variable approach is used to propose harmonic functions, representing out-of-plane displacements, in the form of asymptotic series including both power and logarithmic terms. The procedure has been implemented in computer algebra software Mathematica. A classification of the behaviors of the asymptotic series covering all the considered corner problems is presented. The procedure can be easily modified to deduce singular solutions in corners with free-spring boundary conditions with varying spring stiffness.

Examples of the asymptotic series are presented to illustrate the capabilities of this procedure and the code. A particular case of the corner with the inner angle $\omega=180^{\circ}$, giving, e.g., the upper half plane ($y>0$), is studied in more detail, because it can represent the crack-tip solution for a crack bridged by linear elastic springs under antiplane strain assumptions. In fact, both the even and odd parts of a bridged-crack solution, with respect to the x-axis ($y=0$) can be deduced by the present procedure. The relationships of the asymptotic series deduced for the bridged-crack tip solution with some well-known solutions in fracture mechanics are discussed.

To check the validity of some general conclusions obtained by the previous analysis of the antiplane strain case, a numerical study is carried out using the FEM code Abaqus for a crack bridged by linear elastic springs with varying stiffness under plane strain assumptions. User subroutine UINTER is used for spring distribution modeling. Some theoretical predictions obtained by analyzing a bridged crack in the antiplane strain case have been confirmed by this numerical study for the plane strain case as well.

Finally, the potential of applying the above results in the computational implementation of the Coupled Criterion of Finite Fracture Mechanics by minimizing the total energy with a stress constraint, is explored, and discussed.

3 Conclusions

The presented singularity analysis of corners and cracks with spring boundary condition with varying stiffness opens new possibilities in fracture mechanics, especially in the framework of a promising general computational implementation of the Coupled Criterion of Finite Fracture Mechanics.

References

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Acknowledgements

The research was conducted with the support of Spanish Ministry of Science and Innovation: PID2021-123325OB-I00, Spanish Ministry of Science, Innovation and Universities: PGC2018-099197-BI00; Consejería de Transformación Económica, Industria, Conocimiento y Universidades, Junta de Andalucía: P18-FR-1928, US-1266016; European Regional Development Fund: PGC2018-099197-B-I00, P18-FR-1928, US-1266016.

This research has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 861061—NEWFRAC Project.