

Griffith Fracture in Viscoelastic Elastomers Done Right

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Abstract

Strikingly, “pure-shear” fracture tests have repeatedly shown that fracture nucleation in (common hydrocarbon and other types of) viscoelastic elastomers occurs at a critical stretch that is independent of the stretch rate at which the test is carried out. In this presentation, we will show that this remarkable — yet overlooked — experimental finding implies that the Griffith criticality condition that governs nucleation of fracture from large pre-existing cracks in viscoelastic elastomers can be written in fact as an expression *not* in terms of an elusive loading-history-dependent critical tearing energy T_c , as ordinarily done, but as one exclusively in terms of the intrinsic fracture energy G_c of the elastomer.

1. Introduction and main result

Following in the footsteps of Griffith [1], Rivlin and Thomas [2] famously identified three types of tests — the so-called “pure-shear”, single-edge-crack-tensile, and “trousers” tests — that allowed for an expedient study of when fracture nucleates from large pre-existing cracks in elastomers subjected to quasi-static deformations. While Rivlin and Thomas’ analysis of these tests made critical use of the idealization that elastomers are purely elastic solids, the flurry of subsequent investigations that their pioneering work triggered extended the same approach to account for the fact that elastomers are inherently viscoelastic solids. In so doing, they established that fracture may nucleate from a large pre-existing crack in an elastomer whenever the change in *total* deformation (stored and dissipated) energy \mathcal{W} in the bulk with respect to an added surface area to the pre-existing crack Γ_0 reaches a certain critical tearing energy T_c characteristic of the elastomer:

$$-\frac{\partial \mathcal{W}}{\partial \Gamma_0} = T_c. \quad (1)$$

In this expression, the added surface area refers to the undeformed configuration and the derivative is to be carried out under fixed boundary conditions on the parts of the boundary which are not traction-free. Notably, T_c is *not* a constant but — much like \mathcal{W} — a function of the loading history. Physically, T_c describes the *total* energy (per unit fracture area) expended in the tearing process and thus it contains contributions from the actual creation of new surface as well as from the viscous dissipation (assuming that there are no other dissipation mechanisms, such as strain crystallization) taking place around the crack front and the rest of the bulk.

Experiments carried out at extremely low loading rates, at high temperatures, and on solvent-swollen specimens, when viscous effects are minimized, have shown that

$$T_c = G_c \in [10, 100] \text{ N/m} \quad (2)$$

where G_c denotes the intrinsic fracture energy, or critical energy release rate, associated with the creation of new surface in the given elastomer. It is a material constant, independent of time. Its value is in the same range (2) for all common hydrocarbon elastomers.

More generally, experiments carried out at various loading rates, when viscous effects are not negligible and could even be dominant, have shown that

$$T_c = G_c(1 + f_c), \quad (3)$$

where f_c is a non-negative function of the loading history that scales with the viscosity of the elastomer at hand.

Because the critical tearing energy T_c depends on how loads are applied in time through f_c , the criticality condition (1) is of little use unless f_c happens to be known for the specific loading of interest.

The main result. In this presentation, we will show that the Griffith criticality condition (1) can be reduced in fact to a more useful and fundamental form that involves *not* the elusive critical tearing energy T_c but only the intrinsic fracture energy G_c of the elastomer. We will do so by combining two elementary observations.

Acknowledgements

This work was supported by the National Science Foundation through the Grant CMMI-1901583. This support is gratefully acknowledged.

References

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- [2] Rivlin, R.S., Thomas, A.G., 1953. Rupture of rubber. I. Characteristic energy for tearing. *Journal of Polymer Science* 10, 291–318.