# CYCLIC EFFECTIVE NEAR-FIELD LOADING BASED ON THE DOMAIN INTEGRAL METHOD

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#### Abstract

This paper presents a modification of the domain integral method for cyclic loading and crack closure to compute the cyclic effective J-Integral as a near-field loading parameter. The path-dependency of the solution is discussed for different reference states of the field quantities in a cycle. It turns out that refering to the crack opening time point the cyclic effective J-Integral is path-independent for a domain outside the active plastic zone. The validity of this procedure is discussed by comparison with a global energy approach and theoretical field solutions for the J-controlled zone.

## 1. Introduction

A crack growth law like the Paris-law allows for the computation of the number of cycles for a certain crack extension. Therefore, an appropriate measure for the near-field loading around the crack tip under cyclic loading and consideration of crack closure is necessary. Depending on the loading conditions the scale of yielding near the crack tip may be smaller or larger and thus the applicability of various intensity factors for the near-field becomes more or less justified. The principles of linear elastic fracture mechanics with the cyclic stress intensity factor  $\Delta K$  in combination with the consideration of crack closure is well known and accessible by analytical equations, e.g., Newman (1984). In case of the principles of elastic-plastic fracture mechanics this is not the case.

To overcome this lack a global energy approach can be used to compute a cyclic effective J-Integral. Here, the difference in energy between two crack depths a and  $a+\Delta a$  is considered. As in many references the physical meaning of such a parameter is not clear and it is questionable, if it describes the local situation in the vicinity of a cracktip, e.g., Dowling and Begley (1976), Banks-Sills and Volpert (1991). Thus, the approach based on the domain integral method was followed to determine an intensity parameter for the near-field of a cyclically loaded crack with consideration of crack closure. This paper describes the simulation and evaluation procedure and discusses the path-independency of the cyclic effective J-Integral solution. A proof of consistency with known solutions for the field quantities is given. Finally, the method is used to validate the global energy approach and to show some limitations.

## 2. Simulation procedure

The cyclic elastic-plastic simulation of a growing crack is conducted for a two-dimensional model of a central crack in a plate Fig. 1 a). The boundary conditions at the nodes along the crack path are released after every second cycle to simulate crack advance. To reduce local ratcheting and achieve a stabilized plastic wake to consider the effect of plasiticty induced crack closure a material model with moderate linear kinematic hardening is used. More details are given in Garnadt et al. (2022). All necessary field quantities along various domains around the crack tip are stored and used to compute the cyclic effective J-Integral. Therefore, the known formulation of Shih et al. (1986) is used with a variant definition of the field quantities following the idea of Lamba (1975). All of them are refered to certain time points in the cycle like the crack opening time point or the time point of the upper load reversal, Fig. 1 b), to achieve a cyclic value of the J-Integral and to be able to consider the impact of crack closure. The global energy approach is based on the global force displacement curve and the same characteristic referencing points.

## 3. Results

It turns out that the modified domain integral method with a reference of all field quantities to the time point of crack opening yields a path-independent solution for the cyclic effective J-Integral for the loading branch, Fig. 1 c). Integrating the domain integral for the field quantities refered to the upper point of load reversal up to the time point of crack closure yields a path-independent solution for the unloading branch,

Fig. 1 c). The active plastic zone as indicated in Fig. 1 a) needs to be enclosed by the domain otherwise the result will be path-dependent. Fig. 1 c) as an example shows furthermore, that the result of the domain integral method is consistent with the global energy approach and lies in between the solutions for the energy difference between a- $\Delta a$  and a (-) and a and a+ $\Delta a$  (+). The cyclic effective J-Integral for the loading branch is lower than for the unloading branch, which depends on the loading level.

Finally, the local approach has some advantages over the global energy approach. It is not necessary to choose a unique global control point, which becomes especially important, if the investigated geometry gets more complicated. The solution is available by a relatively coarse mesh and just one computation per crack depth is necessary.



Fig.1 – (a) FE model of a central crack in a plate with boundary conditions and the different plastic zones,
(b) schema of the referencing of all field quantities to characteristic time points in the cycle,
(c) exemplary results of the cyclic effective J-Integral over the number of contours.

## 4. Conclusions

The local approach for determining the cyclic effective J-Integral based on the domain integral method shows a path-independent result for the loading and unloading hysteresis branch, if all field quantities are referred to the crack opening time point for the former and the upper point of load reversal for the latter. The integration limit for the unloading branch is the time point of crack closure. The method is consistent with the global energy approach but has some advantages concerning the applicability.

#### Acknowledgements

This research was financially supported by the Research Association for the Iron and Metal Processing Industry eV (AVIF), Germany. The support of the Research Association for Combustion Engines eV (FVV), the working group "W10 High-temperature Behavior" and the project chairman Dipl.-Ing. Henning Almstedt (Siemens Energy) is gratefully acknowledged.

## References

J. C. Newman. A crack opening stress equation for fatigue crack growth. International Journal of Fracture 24, pages R 131–R 135, 1984.

N. Dowling and J. Begley. Fatigue Crack Growth During Gross Plasticity and the J-Integral. Mechanics of Crack Growth, ASTM STP 590, pages 82–103, 1976.

L. Banks-Sills and Y. Volpert. Application of the cyclic J-integral to fatigue crack propagation of Al 2024-T351. Engineering Fracture Mechanics, 40(2), pages 355–370, 1991.

F. Garnadt, C. Kontermann, and M. Oechsner. Crack Closure in Cycles with Dwell Times at High Temperature, Engineering Fracture Mechanics, 268, 108463, 2022.

C. F. Shih, B. Moran, and T. Nakamura. Energy release rate along a three-dimensional crack front in a thermally stressed body. International Journal of Fracture, 30(2), pages 79–102, 1986.

H. S. Lamba. The j-integral applied to cyclic loading. Engineering Fracture Mechanics, 7(4), pages 693–703, 1975.