REGULARIZATION OF DAMAGE AND FAILURE USING A NON-LOCAL HARDENING VARIABLE IN AN EULERIAN FORMULATION OF INELASTICITY

Martin Kroon^{1*}, MB Rubin²

¹Linnaeus University, Växjö, Sweden ²Technion Israeel Institute of Technology, Haifa, Israel * Presenting Author email: martin.kroon@lnu.se

Abstract

It is known that damage or inelastic softening can cause an ill-posed problem leading to localization and mesh-dependence in finite element simulations. Here, a nonlocal hardening variable is introduced in a finite deformation Eulerian formulation of inelasticity. This nonlocal variable is defined over an Eulerian region of nonlocality, which is a sphere with radius equal to a characteristic length, defined in the current deformed geometry of the material. The influence of the nonlocal hardening variable is studied using an example of a plate that is loaded by a prescribed boundary displacement causing formation of a shear band. Predictions of the applied load vs. displacement curves and contour plots of the total distortional deformation of the plate and the hardening variable are studied. It is shown that the characteristic material length controls the structure of the shear band developed in the plate.

1. Introduction

The microstructural origins of damage localization are, to a large extent, still unclear, and how to model these phenomena is also an open issue. Physically, damage, which causes a reduction in the load carrying capacity of a material, typically is due to local inhomogeneities at the microscale which can cause stress concentrations. This damage can be initiated by breakage of atomic bonds; nucleation, growth and coalescense of pores; or inhomogeneities of mass or dislocation densities. Damage can lead to macroscopic weakening of the elastic material properties or mechanical or thermal reduction of the yield strength. Moreover, these softening mechanisms can be enhanced by geometric inhomogeneity, like that in necking and shear banding, to cause localization of the damaged region. Modelling and simulating damage and softening leading to localization in materials remains a challenge to both theoretical and computational mechanics. Boundary value problems with materials that exhibit strain softening become ill-posed, and finite element analyses of such problems become intrinsically mesh-dependent. These problems can be regularized by the introduction of a characteristic length. One way to do this, is to let the size of the elements in the localization zone in the finite element analysis define the length of the problem. In this sense, the size of a finite element introduces a characteristic length which defines the region of interaction of material points within the element. However, this characteristic length changes with mesh refinement and is not a length characterized by the material. Thus, a better approach is to include the characteristic length in the constitutive model. In the present work, an Eulerian formulation of inelasticity is used, where the evolution of an elastic deformation tensor is prescribed and where this elastic tensor determines the stress in an elastically isotropic material. Here, use is made of a phenomenological theory of elastic-plastic response of metals to develop a simple non-local model to control localization. The formulation does not require additional balance laws or boundary conditions. Specifically, a constant characteristic length is introduced to define an Eulerian nonlocal region of influence, and a nonlocal strength is introduced. Two different definitions of this non-local strength are explored.

2. Results

Fig. 1 shows a plate with a zero-stress thickness of 2 mm, a length of 30 mm, and a width of 10 mm. The plate is loaded in tension and by prescribed displacement, such that the vertical edges remain vertical, as

indicated in Fig. 1. The initial strength of the elliptical region at the centre of the plate is slightly lower than for the rest of the plate, see Fig. 1.



Fig.1 – Geometry and boundary conditions for a plate loaded in tension.

The evolution of a shear band during stretching of the plate is now considered. The plate is discretized using wedge elements, such that the model has one element through the thickness, and in the plane, the element has a triangular profile. Different discretizations are considered, where the characteristic dimension of the finite elements in the plane is denoted by L_e . Three levels of refinement are considered: $L_e = 0.5$ mm, $L_e = 0.25$ mm, and $L_e = 0.125$ mm. Contour plots of the total distortional strain variable ε are used to illustrate localization.

In Fig. 2 the distributions of ε for two nonlocal models and three different mesh refinements are shown. In this case, the remote stress is approximately 1020 MPa. Both models show nice convergence properties, i.e. the three meshes give virtually the same predicted distributions of ε .



Fig.2 – Shear bands predicted by the two different models for different mesh refinements at the post-peak stress 1020MPa.