Griffith Fracture Theory for the Size Effect on Strength of Brittle Materials

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Abstract

The scale- or the size-dependence of mechanical strength in many brittle materials appears to follow a 'universal law,' of the form: strength $\propto L^{-n}$ or V^{-n}, where n is a number, L is the length and V is the volume of the specimen or structure. Broadly known as the "size-effect" in geology, civil engineering, mining and materials science, this behavior determines the strength of large structures such as ice sheets, rock formations, coal pillars in mines and concrete beams and columns in civil infrastructure. As of now, there is no reliable scientific basis or theory to explain the size effect or for determining a reliable value of 'n'. This has been the missing link in strength of materials for nearly a century since the Griffith's crack theory Here, we show that the change in net-section strain energy, due an initial crack in a structure, and its dissipation within a crack layer of finite thickness, leads to the necessary and sufficient physical basis to explain the size-dependence of strength as $\propto L^{-0.5}$. Further, size-independence of strength is explained simultaneously when the crack layer volume approaches the specimen volume. Extensive validation using experimental data from over twenty different studies of ice, sandstone, coal, concrete is demonstrated here to show that the proposed approach provides accurate strength predictions, universally, for large structural scales. By including size in brittle fracture strength prediction, this work significantly advances fracture mechanics beyond the Griffith crack theory.

1. Introduction

Leonardo da Vinci was probably the first to investigate the size-effect on strength of materials. Extensive experimental data is now available to indicate that brittle fracture strength is dependent on the absolute size, specifically in ice, ice sheets, concrete, rock, coal pillars and large civil structures. The data clearly indicate that laboratory scale strength tests severely overestimate the stresses that cause brittle fracture in large-scale structures. The results are often loosely interpreted based on Weibull's statistics of strength which implies that in larger structures, there is greater chance of finding a larger critical flaw for fracture. On the one hand this requires that flaw size to scale with the structure size, for which there is no clear experimental evidence. On the other hand, the statistical theory is disputed by experimental data showing strong size-dependence of strength even in notched specimens where the flaw size is fixed and is independent of specimen size. The correlations known as size-effect-laws do not explain, from physical principles, why strong size-effects are observed universally across geomaterials. A physical basis to explain the size-effect, is developed in this work.

2. Results

A physically based size-strength relationship for brittle materials can be derived in the form of a Griffith crack theory, by considering *the change in net-section strain energy* when the crack is introduced in a crack-free solid. The idea is that the change in strain energy stored in net-section volume alone is sufficient to cover the energy required for fracture in the crack layer of a brittle material. Referring to Figure 1, consider a specimen with a crack of size, a, elastically deformed under a load, P. At the point of fracture, the maximum elastic strain energy, contained in the net-section volume, consists of two parts--one is the strain energy that existed in that volume due to P, but before the crack was introduced, and the other is the increase contributed by the strain energy released from portions above and below the crack (ABCD; Fig. 1(a)). This increase is termed as *the change in net-section strain energy* and, by Clapeyron's theorem, it is exactly equal to the work done at the net-section boundary due to crack introduction. At the point of fracture, the total strain energy stored in the net-section is spent in two ways—the first part is the elastic recovery or unloading of material volume of the net-section (Fig. 1b), including the unloading of the fragments created within the crack layer, and the second part is the energy dissipated in crack layer in the actual fracture process (Fig. 1c), which may include micro-cracking, nonlinear deformation, and friction between fragments within the crack layer. Since the first part must be equal to the strain energy that existed in net-section before the crack was introduced, the *change in net-section strain energy* is that which is exactly consumed in the fracture process.

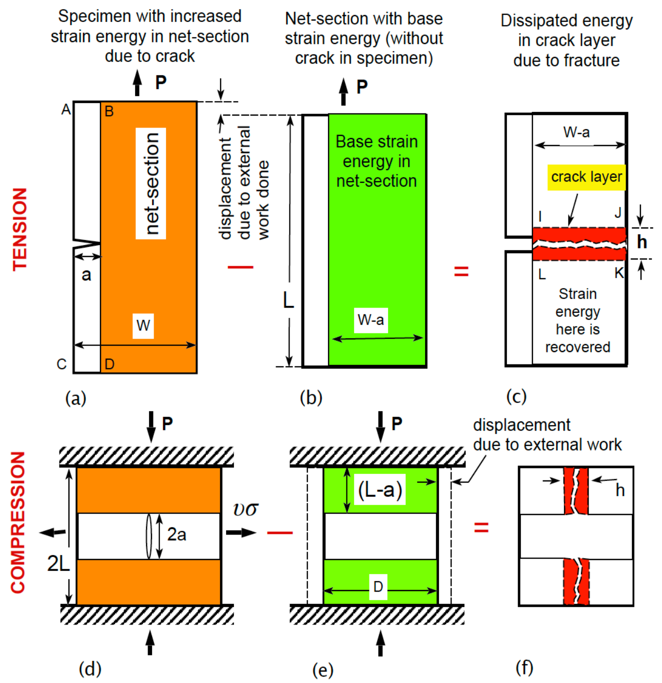

Figure 1

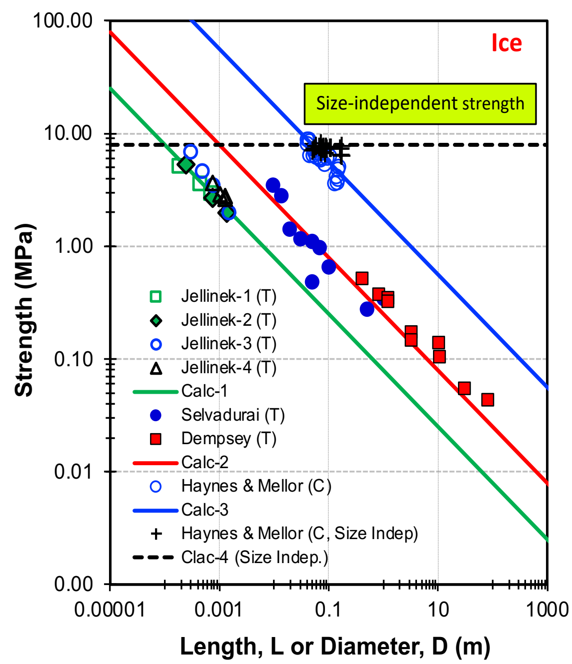

Figure 2

Figure 1. The concept of the change in net-section strain energy. For tension loading: (a) net-section with an increased strain energy after introducing the crack, (b) strain energy of the net-section before crack introduction and (c) the energy spent in crack layer. (d-f) equivalent situations under compression.

Figure 2. The confirmation of $L^{-0.5}$ or $D^{-0.5}$ size-dependence of strength for ice

Based on the above theory and after some derivation using essential mechanics, the brittle fracture strength, σ_f, can be written as

$$\sigma_f = \sqrt{2E\gamma_f}\sqrt{\frac{h}{L}}\left[\frac{(1-a/W)}{\sqrt{a/W}}\right]$$

The above equation reveals that for tension specimens of varying length, *the fracture strength* is proportional to $L^{-0.5}$. Figure 2 shows the size-dependence of fracture strength of ice, as predicted by the above equation and comparison with experimental results on strength of ice, as determined from specimens of different sizes, by several investigators.

3. Conclusions

A physical basis for the size-dependence of brittle fracture strength in geomaterials, while also predicting the size-independent behavior, has been demonstrated based on the idea of change in net-section strain energy. The theory presented here predicts universally that brittle fracture strength $\propto L^{-0.5}$. This represents a significant advance beyond Griffith crack theory, in that the form of size-dependence naturally emerges from the present theory, and it reduces to the Griffith's crack theory as the size-independent theory.