

**A Direct Approach to Fatigue Crack Growth Under Large Scale Plasticity  
(Presentation in honor of JoDean Morrow, University of Illinois)**

K. S. Ravi Chandran<sup>1</sup>

<sup>1</sup>University of Utah, Department of Materials Science and Engineering, Salt Lake City, USA

\* Presenting Author email: ravi.chandran@utah.edu

**Abstract**

The major challenge in the mechanics of elastic-plastic fatigue crack growth (FCG) is to find a physically based driving force to correlate the crack growth rates under stress-controlled and strain-controlled conditions. Specifically, a parameter capable of providing a *single-valued* correlation of crack growth rate, regardless of applied fatigue stress/strain values, is needed. Approaches of the past used either cyclic strain (strain intensity factor) or nonlinear fracture mechanics based (cyclic J-integral,  $\Delta J$ ) parameter, to correlate fatigue crack growth. The latter, however, requires experimental load-deflection curve after every crack length increment and geometry correction factors, which are complex. In the present work, it is shown that a new and physically based approach, based on the *cumulative change in the cyclic strain energy of the net-section*, is used to successfully correlate fatigue crack growth in a variety of loading elastic-plastic loading situations. The change in the cyclic strain energy is determined analytically from tensile elastic-plastic behavior of material and from the relative sizes of cracked and uncracked sections in the crack plane. Remarkably, excellent correlations of fatigue crack growth data in a variety of specimen geometries and stress/strain levels have been found for both *stress- and strain-controlled fatigue* conditions. This work, in conjunction with the author's earlier works, validates that the change in net-section strain energy is a fundamental quantity in the mechanics of fatigue crack growth.

**1. Introduction**

In major engineering applications (high temperature components in thermal/nuclear power plant piping, pressure vessels, turbine blades and rotors, solder joints) growth of fatigue cracks are accompanied by large deviations from linear elasticity, either in the form of large plastic zones at the crack tip or the plastic/nonlinear deformation of the net-section. The characterization of crack growth in these conditions, which are referred to as, "elastic-plastic crack growth conditions," are challenging, due to the nonlinear deformation conditions. J. R. Rice had stated (in his 1975 paper), "There are as yet no precise analyses of elastic-plastic fields for cyclic loading applications as in fatigue..." referring to the difficulty in relying on  $\Delta J$  as driving force for fatigue crack growth.

**2. Results**

A new physical approach for direct correlation of FCG both for stress-controlled and strain-controlled fatigue is presented here. It is demonstrated here that excellent correlations of fatigue crack growth, under a wide range of crack sizes, stress/strain levels, specimen geometries and materials, can be achieved using the concept of the change in net-section strain energy that occurs during crack growth. The concept of the change in net-section strain energy, at a given crack size, is illustrated in Figure 1 for strain-controlled cyclic loading. Here, crack propagation occurs with remote tension/compression strains controlled at the specimen boundaries. Here, the change in net-section strain energy between a crack-free specimen (Figure 1(a)) loaded to a plastic displacement ( $\Delta_\infty$ ) and that having a crack length,  $a$ , loaded to the same displacement (Figure 1(b)), is given by the area between the two load-displacement curves shown in Figure 1(c). The deformation state of the net-section crack plane is illustrated in Figure 1(d). The change in strain energy of the net-section, relative to the crack free specimen can be calculated (after some steps) as

$$\Delta C_{p\epsilon} = \frac{K[\epsilon_{p\infty}^{(n+1)} - \epsilon_o^{(n+1)}]}{(n+1)} \left[ \frac{a}{(W-a)} \right] \quad \text{for strain-controlled elastic-plastic crack growth} \quad (1)$$

$$\Delta C_{p\sigma} = \frac{n[\sigma_\infty^{(n+1)/n} - \sigma_o^{(n+1)/n}]}{K^{1/n(n+1)}} \left[ \left\{ \frac{W}{W-a} \right\}^{1/n} - 1 \right] \quad \text{for load-controlled elastic-plastic crack growth} \quad (2)$$

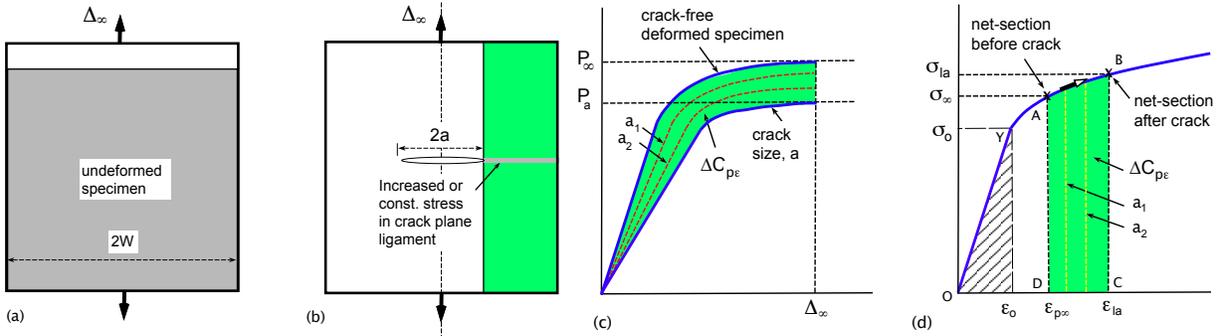


Figure 1. The change in net-section strain energy in strain-controlled fatigue; (a) specimen without a crack after deformation under remote displacement, (b) specimen with a crack deformed to the same displacement, (c) the change in net-section strain energy in the global sense and (d) the elastic-plastic deformation of the crack plane in net-section

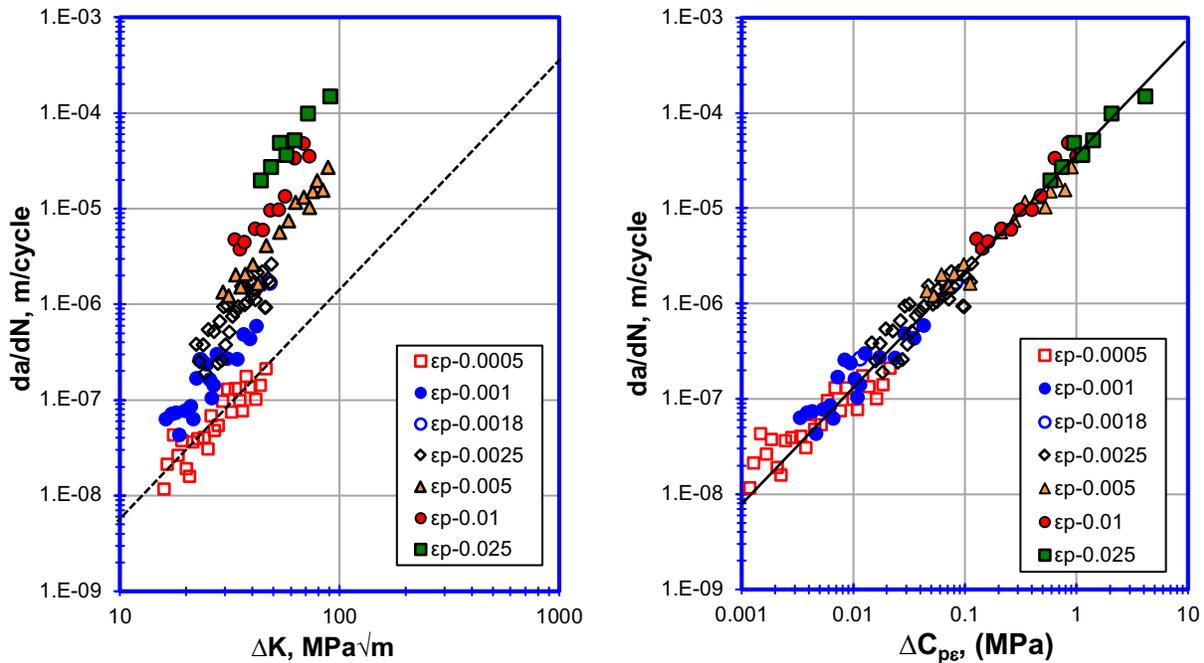


Figure 2. Fatigue crack growth under strain control for 1018 Steel, for SENT specimens, plotted in terms of (a)  $\Delta K$  and (b) the change in net-section strain energy,  $\Delta C_{pe}$ . (Data from Solomon).

where  $\sigma_\infty$  and  $\epsilon_{p\infty}$  represent the applied cyclic stress or strain amplitudes, respectively,  $\sigma_0$  and  $\epsilon_0$  are the yield stress and strain, respectively,  $a$  is the crack length and  $W$  is the width of the specimen or structure. The other parameters ( $K, n$ ) are the well-recognized material stress-strain parameters. Figure 2 demonstrates the excellent correlations of FCG data, obtained under strain-controlled fatigue testing using equation (1).

### 3. Conclusions

It is shown that, using only the material strain hardening parameters ( $K, n$ ), crack size ( $a$ ) and specimen width ( $W$ ), crack growth behavior under large-scale plasticity can be correlated very well based on the change in net section strain energy. The change in net-section strain energy approach is validated for both stress- and strain-controlled testing by using the raw crack length data for various materials and specimen geometries, acquired over wide ranges of applied stresses and strains.