I-INTEGRAL FOR MAGNETO-ELECTRO-ELASTIC MATERIALS WITH RESIDUAL STRAIN

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Abstract
Magneto-electro-elastic (MEE) materials generally consist of piezoelectric and piezomagnetic constituent phases so that the residual strain often occurs during the manufacturing process. The distributed residual strain and complex interfaces between constituents cause a great challenge to the fracture analysis of MEE materials. Considering the effect of the residual strain, this paper develops an interaction integral (I-integral) method for the extraction of the fracture parameters of an impermeable crack in nonhomogeneous MEE materials.

1. Introduction

With the development of modern society, smart materials are being widely applied in various engineering areas, including aerospace, health monitoring, civil engineering and nanoelectromechanical systems. However, residual strain in MEE materials generated during thermo-mechanical processing usually enhances crack propagation. Therefore, a thorough interpretation of the fracture mechanism of MEE materials under residual strain is necessary. Up to now, the I-integral for MEE materials subjected to residual strain has not been reported in available publications.

2. I-integral

The present paper concentrates on an impermeable crack in the MEE materials with residual strains. The I-integral is derived from the J-integral by superimposing an auxiliary state onto an actual state.

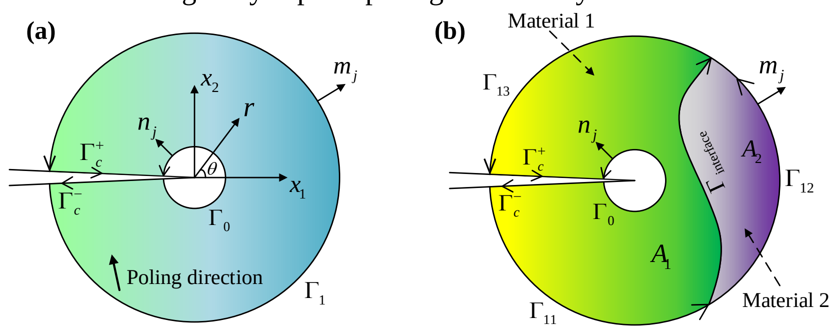


Fig.1 – (a) Schematic diagram of contours and integration domain around the crack tip; (b) an integration domain cut by an interface

As shown in Fig. 1(a), the J-integral for a 2D cracked MEE solid is

$$J = \lim_{\Gamma_0 \to 0} \int_{\Gamma_0} (F\delta_{1j} - \sigma_{ij}u_{i,1} - D_j\phi_{,1} - B_j\varphi_{,1})n_j d\Gamma \tag{1}$$

The I-integral expression of inhomogeneous magneto electro elastic materials considering residual strain is derived as

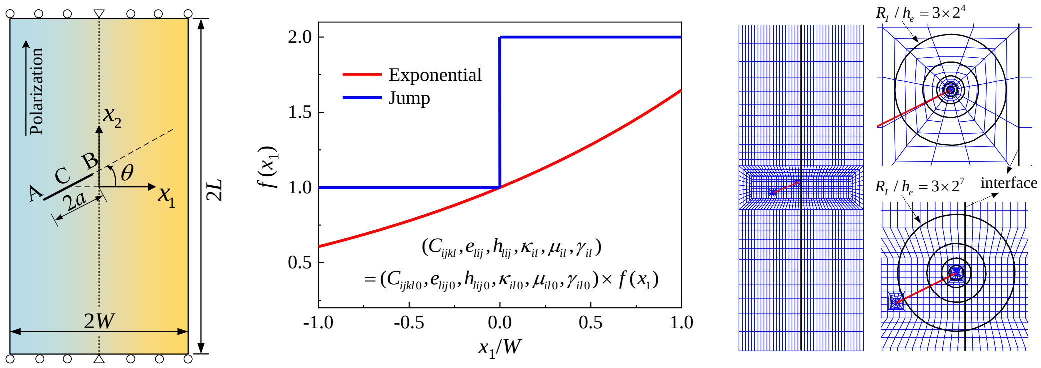
$$\begin{aligned} I = & \int_A \begin{Bmatrix} \sigma_{ij}u_{i,1}^{aux} + \sigma_{ij}^{aux}u_{i,1} - \sigma_{ik}^{aux}\varepsilon_{ik}^{m}\delta_{1j} \\ +D_j\phi_{,1}^{aux} + D_j^{aux}\phi_{,1} + D_i^{aux}E_i\delta_{1j} \\ +B_j\varphi_{,1}^{aux} + B_j^{aux}\varphi_{,1} + B_i^{aux}H_i\delta_{1j} \end{Bmatrix} q_{,j}dA + \sum_{s=1}^{n}\int_{A_s}\sigma_{ij}^{aux}\varepsilon_{ij,1}^{r}qdA \\ & + \int_A \begin{Bmatrix} \sigma_{ij}\left(S_{ijkl}^{tip} - S_{ijkl}\right)\sigma_{kl,1}^{aux} + \sigma_{ij}\left(\eta_{kij}^{tip} - \eta_{kij}\right)D_{k,1}^{aux} + \sigma_{ij}\left(g_{kij}^{tip} - g_{kij}\right)B_{k,1}^{aux} \\ +D_j\left(\eta_{jkl}^{tip} - \eta_{jkl}\right)\sigma_{kl,1}^{aux} - D_j\left(\beta_{jk}^{tip} - \beta_{jk}\right)D_{k,1}^{aux} - D_j\left(\alpha_{jk}^{tip} - \alpha_{jk}\right)B_{k,1}^{aux} \\ +B_j\left(g_{jkl}^{tip} - g_{jkl}\right)\sigma_{kl,1}^{aux} - B_j\left(\alpha_{jk}^{tip} - \alpha_{jk}\right)D_{k,1}^{aux} - B_j\left(\lambda_{jk}^{tip} - \lambda_{jk}\right)B_{k,1}^{aux} \end{Bmatrix} qdA \end{aligned} \tag{2}$$

Next, we discuss the influence on the interaction integration method when the integration region contains the material interface. As shown in Fig. 1(b), in domain *A*, there is a perfectly bonded interface $\Gamma_{\text{interface}}$ across which the MEE properties jump. Then, in addition to the domain integrals in Eq. (2), a line integral along the interface needs to be calculated.

$$I_{\text{interface}} = \int_{\Gamma_{\text{interface}}} \sigma_{ij}^{aux} \left[\left(\varepsilon_{ij}^{r} \right)^{②} - \left(\varepsilon_{ij}^{r} \right)^{①} \right] m_1 q d\Gamma \tag{3}$$

3. Results

As shown in Fig. 2, a nonhomogeneous MEE plate whose properties vary with x_1 according to $(C_{ijkl}, e_{lij}, h_{lij}, \kappa_{il}, \mu_{il}, \gamma_{il}) = (C_{ijkl0}, e_{lij0}, h_{lij0}, \kappa_{il0}, \mu_{il0}, \gamma_{il0}) \times f(x_1)$ .The exponential function and the jump function are adopted. The plate is of length $2L = 60$ and width $2W = 20$. The crack tips are located at $\mathrm{A}(-4.6, -1)$ and $\mathrm{B}(-0.6, 1)$. The jump residual strain of $\varepsilon_{\mathrm{J}}^{\mathrm{r}}(x_1 \le 0) = -0.0008$ and $\varepsilon_{\mathrm{J}}^{\mathrm{r}}(x_1 > 0) = -0.0002$ is applied to the plate. It can be obtained that the relative deviations $R_d(K_I)$, $R_d(K_{II})$, $R_d(K_D)$ and $R_d(K_B)$ are within 0.04%, 0.08%, 0.07% and 0.26% , respectively, for every material function $f(x_1)$.



(a) (b) (c)

Fig.2 – (a) Geometry and boundary conditions of a cracked plate; (b) material functions; (3) finite element mesh and integration domains cut by an interface

Table 1. Normalized intensity factors at the tip B for different integration domains

R_I/h_e	K_I^*	K_{II}^*	K_D^*	K_B^*
Exponential				
6	1.1548	0.5378	0.7430	0.4585
48	1.1543	0.5376	0.7427	0.4576
384	1.1546	0.5374	0.7430	0.4580
R_d (%)	0.04	0.07	0.04	0.20
Jump				
6	1.1179	0.5191	0.7279	0.4317
48	1.1174	0.5189	0.7274	0.4306
384	1.1177	0.5187	0.7279	0.4312
R_d (%)	0.04	0.08	0.07	0.26

4. Conclusions

Taking into account the effect of the residual strain, a modified I-integral method is proposed to extract the intensity factors (i.e., SIFs, EDIF, and MIIF) of an electrically and magnetically impermeable crack in MEE materials. Moreover, whether the material properties across interfaces are continuous or not, the I-integral is domain-independent for nonhomogeneous and multi-interface materials as long as the residual strain is continuous in the integration domain. When the residual strain is discontinuous across the interface, the line integral along the interface is usually non-zero and needs to be calculated in the I-integral calculations.

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