Adaptive Multiple Importance Sampling for Structural Risk Assessment

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Abstract

The USAF Airworthiness Bulletin (AWB)-013A, *Risk Identification and Acceptance for Airworthiness Determination* defines airworthiness in terms of the probability of aircraft loss per flight hour[1] with one important component being the aircraft structure. The probability-of-failure of an aircraft component is challenging to compute due to its small size, typically 10^{-7} or less. As a result, simplified fracture mechanics models are usually used with a small number of random variables. However, these simplifications may lead to an inaccurate probability-of-failure estimate. To address this issue, an adaptive multiple importance sample method was developed that can compute very low probabilities with significant efficiency. This allows one to consider more realistic fracture mechanics models and a larger number of random variables than has been previously possible. The method is adaptive in that it will adjust to the varying relative importance of the random variables for different applications. Convergence is ensured such that the coefficient of variation is below a user-defined threshold. A significant side benefit is that AMIS can also be used to determine an optimized inspection schedule with a small number of additional fracture analyses. Results to date show efficiency gains of 5 or 6 orders of magnitude over standard Monte Carlo sampling for typical problems of interest. The methodology will be outlined and demonstrated using aircraft example problems.

1. Introduction

A probabilistic damage tolerance analysis software program Smart|DT was developed under FAA funding. This software is a comprehensive tool that considers fracture mechanics crack growth models, loading, material properties, geometric properties, and inspection and repair, to computer the probability-of-failure. A critical feature of the software is the ability to compute the probability-of-failure efficiently. Traditional sampling methods are far too inefficient. As an example, for a probability-of-failure of 10^{-7}, approximately 10^{9} samples are required for standard Monte Carlo[2]. Other methods such as numerical integration are efficient only for a small number of random variables, e.g., 3-4. Therefore, an adaptive multiple importance sample (AMIS) method was developed that can compute very low probabilities with significant efficiency compared to standard Monte Carlo yet can be expanded to consider a larger number of random variables compared to numerical integration[3].

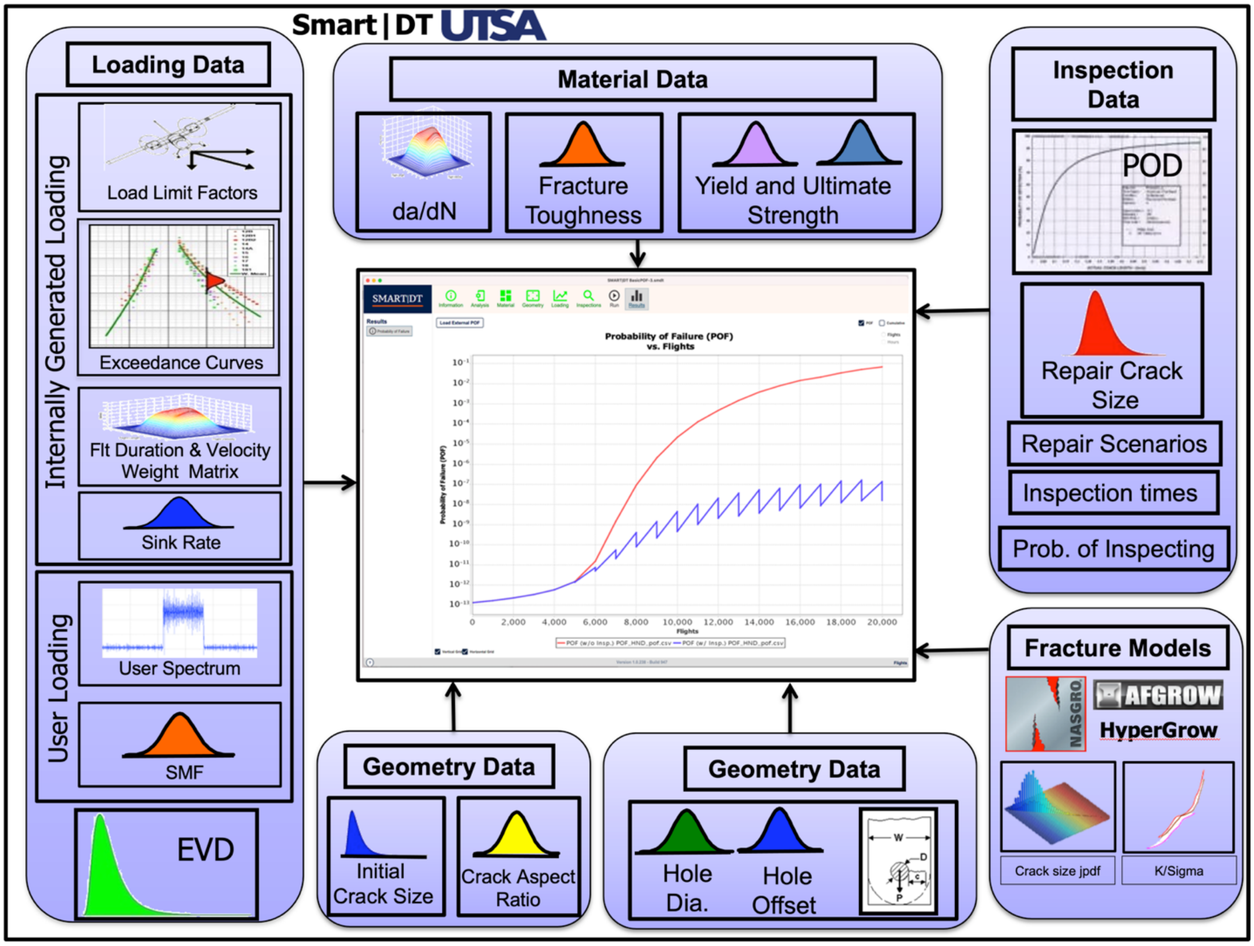

Figure 1 Overview of the Smart|DT software

The most fundamental aspect of AMIS is that it will detect the important variable values that contribute the most to the probability-of-failure. In addition, since the probability-of-failure is needed at multiple flight hours (say every 1000 hours), a mixture density is developed across all times requested by the user that is a weighted mixture of densities. As an example, Figure 2 shows the contours for the joint probability density function with respect to crack size and fracture toughness for a specific application.

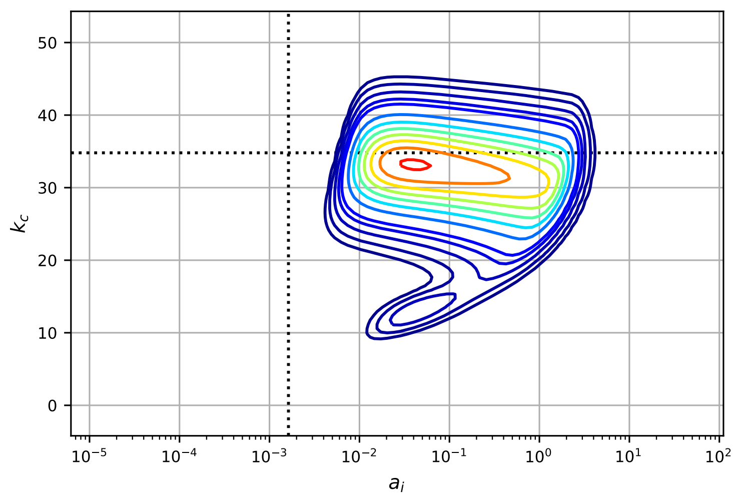

Figure 2 Example of multiple density results

2. Results

Figure 3 shows the probability-of-failure results for an example problem of a crack growing from a hole subjected to variable amplitude loading. The figure shows results without and with inspections for both standard Monte Carlo sampling (10^9 samples) and AMIS ($\sim 10^3$ samples). The bottom portion of Figure 3 shows an estimate for the coefficient of variation (COV) of the probability-of-failure values as a function of flight times. The results indicate that the COV is below 0.1 for all times. Samples are added to the analysis if the COV is larger than the user specified threshold value for any flight hour values. In this example, the efficiency improvement over standard Monte Carlo is approximately 6 orders of magnitude.

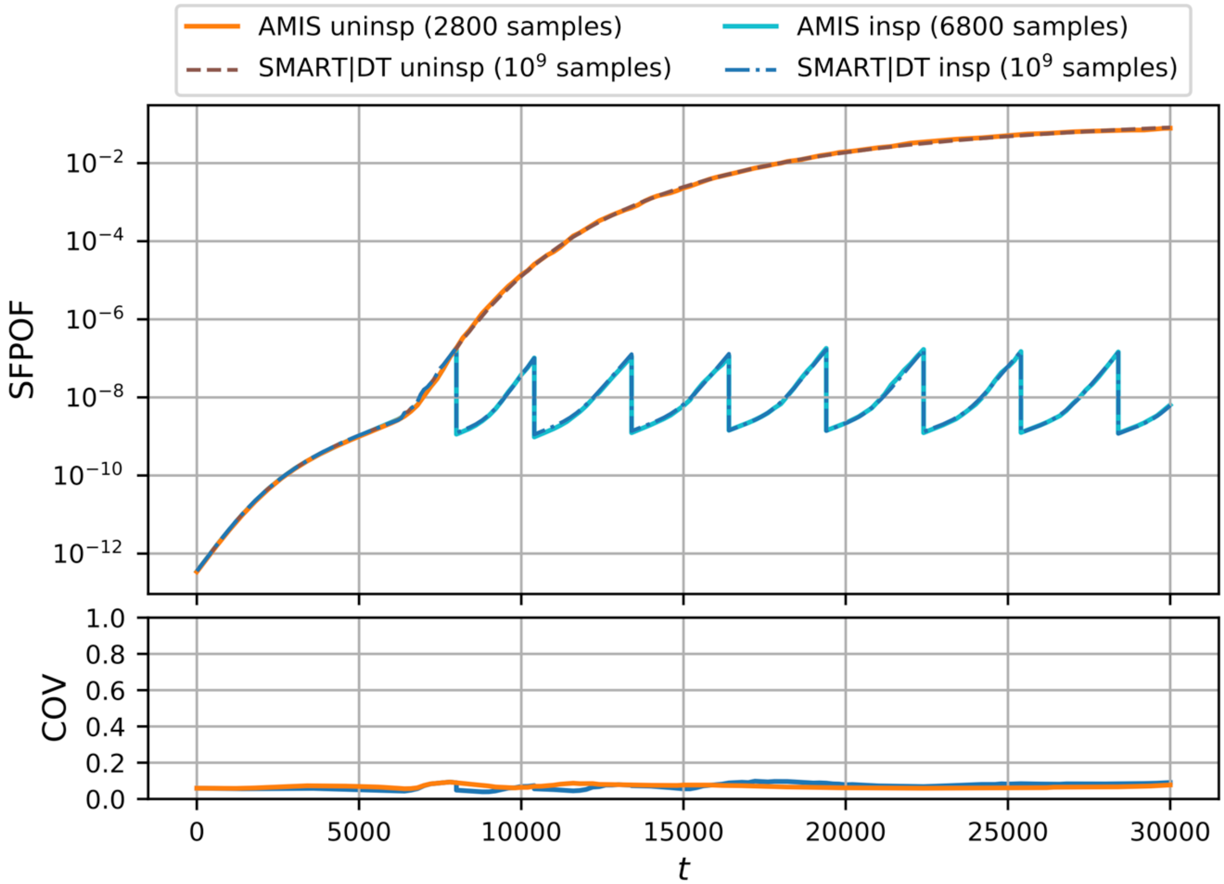

Figure 3 Probability-of-failure results for an example problem of a crack growing from a hole

2. Conclusions

The development of an adaptive multiple importance sampling method provides an efficient method to compute the small probability-of-failure values typical of structural damage tolerance analysis. In addition, the use of sampling, as opposed to numerical integration, provides a means to consider a larger number of random variables such as crack growth variability, hole diameter, and edge distance.

Acknowledgements

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References

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