SINGULAR INTEGRAL EQUATION FOR SOLVING COHESIVE CRACK PROBLEM FOR INITIALLY RIGID TRACTION-SEPARATION RELATION

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Abstract
In case of an initially rigid traction-separation cohesive relation, the total potential energy is not differentiable. This makes the use of variational operator over it questionable. Therefore, the accurate application of FEM is mathematically doubtful. The present work bypasses this issue by modelling the cohesive crack problem as a singular integral equation and solving it using Chebyshev polynomials.

1. Introduction

In order to numerically find the near-tip solution for a cracked solid in the presence of cohesive tractions, the total potential energy is

$$F(u_i) = \frac{1}{2}\int_{\Omega} C_{ijkl} e_{kl} e_{ij} \, d\,\Omega - \int_{S_t} T_i u_i \, d\,S_t - 2\int_{S_{coh}} t_i u_i \, d\,S_{coh}$$

where the first term in the RHS is due to the bulk, the second term due to external boundaries of the solid and the third (last) term due to cohesive tractions on the crack faces. The last term on the RHS may be written as

$$2\int_{S_{coh}} t_i u_i \, d\,S_{coh} = \int_{S_{coh}} d\,S_{coh} \int_0^{u_i} t_i \, d\,2u_i = \int_{S_{coh}} d\,S_{coh} \psi_c$$

where

$$\psi_c = \int_0^{u_i} t_i \, d\,2u_i$$

which for the following initially rigid traction-separation relation

$$t_i = t_c + \alpha 2u_i$$

with α<0 gives a non-differentiable ψ_c as shown in Figure 1 below.

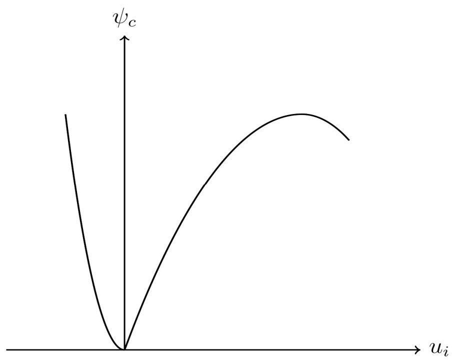

Figure 1 Qualitative variation of ψ_c around the crack tip (u_i=0)

This implies that the total potential energy F is also non-differentiable which makes the application of variation over it questionable. This paper suggests an alternative method to solve the cohesive crack problem for initially rigid traction-separation relation, which does not involve the principle of variations.

This problem is modelled as a singular integral equation and solved numerically using Chebyshev polynomials.

2. Results

The displacement in the crack plane, in the presence of initially rigid traction-separation relation between crack faces, may be modelled as the following singular integral equation

$$u_2(x) = \frac{-1}{2(1-k^2)G}\left\{\frac{1}{\pi}\int_{-a}^{x}\frac{1}{\sqrt{a^2-x^2}}\left[\int_{-a}^{a}\frac{\left(t_c+\alpha 2u_2(\xi)\right)\sqrt{a^2-\xi^2}}{\xi - x}d\xi\right]dx\right\} + \frac{-1}{2(1-k^2)G}\int_{-a}^{x}\frac{\sigma_{yy}^{\infty}x}{\sqrt{a^2-x^2}}dx$$

This maybe numerically solved using Chebyshev polynomials. The normal stress field ahead of the crack tip, for different values of normalized α, may be seen in the figure below.

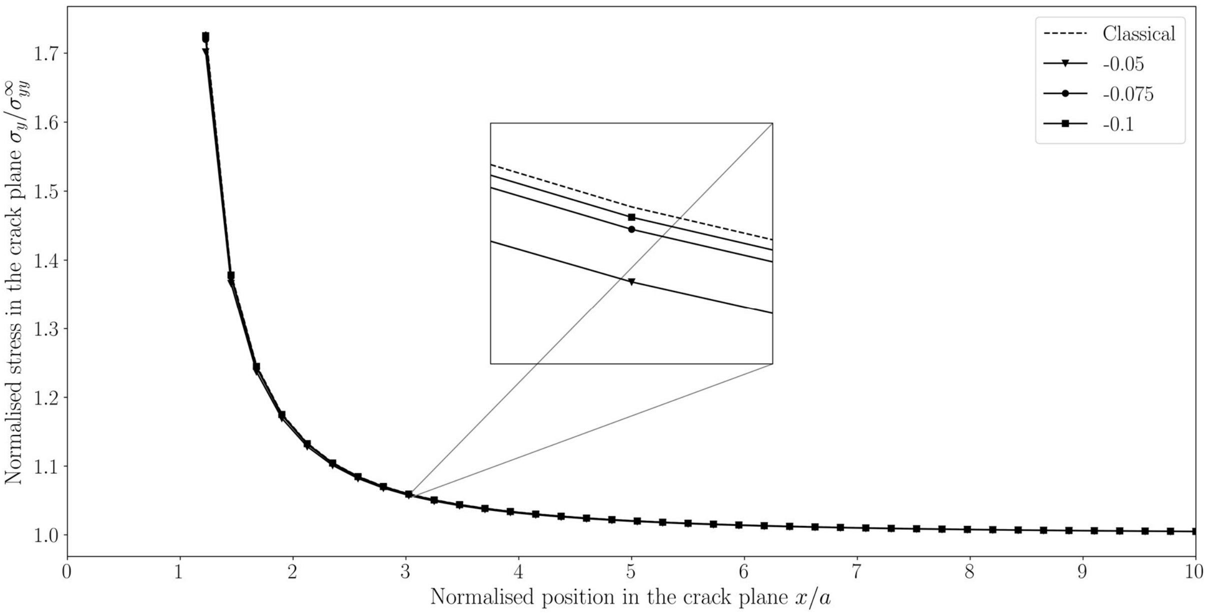

Figure 2 Normalized normal stress ahead of the crack tip for different values of traction-separation relation α. The classical case represents the variation in absence of cohesive stresses

3. Conclusions

The numerical solution of the cohesive crack problem using integral equation, for initially rigid traction-separation cohesive relation, has shown that the presence of the cohesive tractions in crack will make the crack opening more difficult, reduce the singularity in the normal stress ahead of the crack tip. It also gives the cohesive zone as a part of the solution.

Though the integral equation can be used to solve the cohesive crack problem for an initially rigid traction-separation relation, it has it's drawbacks. The major drawbacks are it's inability to handle complicated geometry of the crack, and limitiation for only linear or linear-based (bi-linear, trapezoidal, etc.) traction-separation relations. The latter of the above drawbacks is because of the current state of the arts in applied mathematics as no known methods exist to solve the singular integral equations for non-linear or inverse traction-separation relations. Despite this limitation, it may be argued that singular integral equation is mathematically correct way to solve the cohesive crack problem when the total potential energy is non-differentiable.