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FINITE ELEMENT SIMULATION OF CRACK PROPAGATION IN BRITTLE PLATES

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Abstract

A method for modeling fracture in stiff plates of uniform thickness is presented. The Mindlin-Reissner plate theory within the framework of the finite element method is utilized. The fracture criterion is based on the Rankine theory, in which a crack is initiated when normal traction at a node exceeds a given tensile strength. The traction is calculated as a path integral around a crack tip or a tentative split. The propagation direction is such that the normal traction at the crack tip is maximized. A time-based criterion for crack initiation and propagation is added to the model, which yields better correspondence with the experimentally observed fracture patterns. The proposed methodology was implemented in an in-house code. Initial validation shows excellent agreement between the proposed methodology's predictions and the realistic fracture patterns of ice floes.

1. Introduction

In recent years, substantial efforts have been dedicated to studying ice fracture – a process that can take different forms depending on the scale, geometry, and type of ice. The current work focuses on modeling the brittle fracture of ice floes under bending caused by ocean waves. Ice floes having an approximately uniform thickness can be modeled accurately by relying on plane assumptions (i.e., 2D mesh) in finite element analysis (FEA). This approach has been employed for modeling large-scale ice sheets and wave-ice interactions, and the same assumption is utilized in the current work. The fracture model uses the tensile strength criterion [1], where the fracture initiation relies on maximal normal component of the traction vector $\bar{q}$. At each node of the mesh, a tentative fracture direction is selected, and the traction vector is then computed as a line integral on each side of a tentative split

$$\bar{q}_k = \frac{1}{2}\int_{\partial\Omega_k} \boldsymbol{\sigma}\,\hat{n}\,dS$$

where k=1,2 denotes the side of the split, $\boldsymbol{\sigma}$ is the stress tensor, and $\hat{n}$ is the outward-pointing normal to the integration path (Fig. 1a,1b). The direction of the split is selected in a way that maximizes the magnitude of the normal traction (Fig. 1c), and the calculations are performed for all nodes on both sides of the plate.

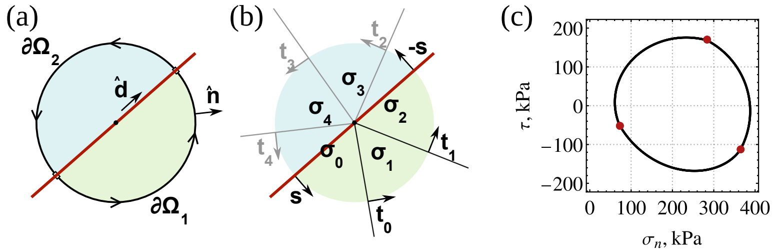

Fig. 1 – (a) Integration contour around the split for normal traction computation, (b) corresponding FEM discretization, (c) normal traction σ_n vs. tangential traction τ for varying angle of the split.

If the normal traction exceeds a predefined tensile strength, i.e., crack initiation criterion, for a prolonged period (e.g., 0.2 s), the corresponding elements are split. Such a time limit allows to resolve the load application by water waves, whose osciallation periods are on the order of one second. At the same time, any short-duration events, such as vibrations of the plate itself, are excluded from the fracture model. After the crack is initiated, the simulation time step is temporarily reduced by a preset factor, i.e., 10^4, to allow sufficient time resolution for crack propagation. This algorithm is executed with an implicit formulation for the equation of motion and implemented in a custom code which can be ported to mainstream FEA codes if needed.

2. Methodology and Results

The bending loads were transmitted to the ice floes via water waves. Two setups were tested, (1) a rectangular floe in a basin with a standing wave and (2) a floe breakup in traveling waves approaching from multiple directions. In the first setup, a rectangular sheet measuring 20x5x0.1 m was loaded with 4 m length waves, an oscillation period of 1.6 s, and an amplitude of 0.1 m. The resulting fracture pattern is shown in Fig. 2 and is consistent with the experimental observations at the initial stages of fragmentation [2]. The width of the formed strips is approximately equal to half of the wavelength, with cracks originating at the peaks of the waves.

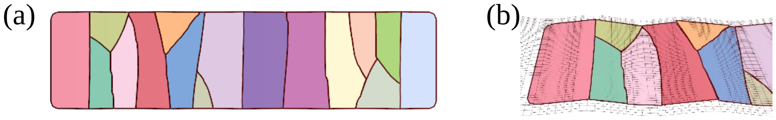

Fig. 2 – Breakup of a rectangular floe after loading in a standing wave, (a) top view, (b) side view.

The second setup models the breakup of a large (landfast) floe under load by traveling waves as shown in Fig. 3. The floe's dimensions are set to 100x50x0.1 m, and the incoming two waves travel at 90 degrees to one another with a 4 m wavelength at a speed of 1 m/s. The waves' amplitude gradually increases to 12 cm over 5 s, then subsides over the same period. Additionally, the amplitude decreases towards the lower part of the floe to imitate a landfast condition. After the splitting completes, the fragments are spread out by a tangential force for better visualization. Mesh resolution is uniform across the entire sheet with an elemental characteristic length of about 15 cm.

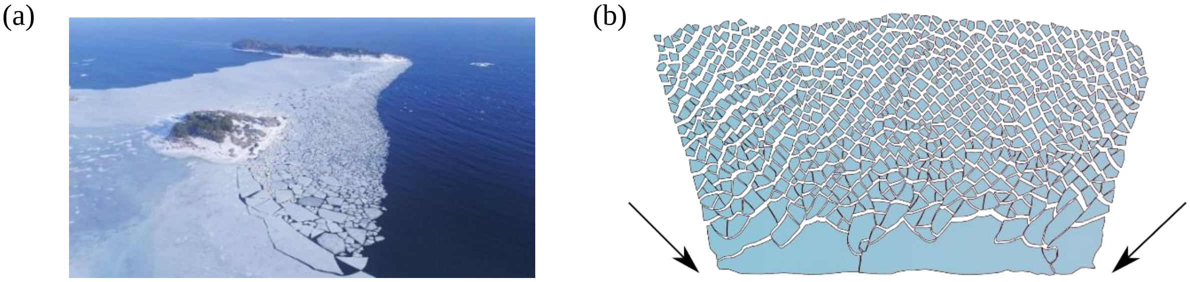

Fig. 3 – (a) Sea ice breakup near Villinki, Finland; (b) simulation result of a floe breakup under the effect of two traveling waves. Arrows indicate the travel direction of the waves.

3. Conclusions

The proposed numerical approach shows an excellent agreement with the observed fracture patterns in natural ice floes and laboratory experiments. The current results are primarily qualitative, but the model shows potential for application in ice mechanics as well as other areas of engineering where bending brittle plates are considered. The formulation is limited to an applicable range of strain rates (currently the wave loading rates), but can be adjusted for a particular engineering application or to consider additional physical phenomena.

Acknowledgements

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