15th International Conference on Fracture (ICF15)
June 11-16, 2023 Atlanta, GA USA

Least Squares Method for mode III SIF Calculation

Jorge Guillermo Díaz Rodríguez[1]*

[1] *Escuela de ingeniería mecánica. Universidad Industrial de Santander, Bucaramanga, Colombia*
** Presenting Author email: jgdiazro@correo.uis.edu.co*

Abstract
The paper presents an experimental approach to determine the stress intensity factors (SIF) of cracks subjected mode III fracture mode. The proposed scheme fits displacement data to Williams series for cracks solving the equations by the overdeterministic Least Squares Methods (LSM). An example is presented for a tube with a through-crack under axial and cyclic torque loading where displacement field measurements were taken with the Digital Image Correlation (DIC) technique. The experimentally determined mode III SIF with the proposed method are compared with respective values found using crack opening displacement (COD) equations. It is concluded that the proposed LSM method agrees with COD measurements but exhibiting less noisy results.

1. Introduction

Although it is agreed that cracks prefer to grow perpendicular to normal stress, cracked bodies can exhibit the three fracture modes depending on the loading. The William´s series, as shown in Eq.(1), describe the displacement field under mode III for an infinite cracked plate with coordinate system located at the crack tip.

$$w = \sum_{n=1}^{\infty} \frac{2r^{n-1/2}}{G} \left\{ c_n Sin\left(n - \frac{1}{2} \right)\theta \right\} \tag{1}$$

where is perpendicular-to-plane displacements, $c_I = K_{III}/\sqrt{2\pi}$, r and θ are radius and angle of an arbitrary point, G is shear modulus, n the numbers of terms in the expansion series, and k is the Kolosov constant. Williams series for mode III can be rewritten as Eq. (2).

$$w = \sum_{n=1}^{N} c_n p_{n(r,\theta)} + T_z + Rz_n \tag{2}$$

where Tz is rigid body displacement in perpendicular-to-plane and Rzn rotation about perpendicular-to-crack-plane axis. An overdetermined system, can be used to solve Eq (2) for mode III as presented in Eq. (3).

$$\{h_{III}\} = [b_{III}]\{\Delta_{III}\} \tag{3}$$

which can be expanded to Eq. (4).

$$h_{III} = \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}; \Delta_{III} = \begin{bmatrix} C_1 \\ \vdots \\ C_M \end{bmatrix}; b_{III} = \begin{bmatrix} p_{1(r_1,\theta_1)} \\ \vdots \\ p_{M(r_M,\theta_M)} \\ 1 \\ 1 \end{bmatrix}^T \tag{4}$$

where M is the number of data points, $[h_{III}]$ is the concatenated vector of displacements, $[\Delta_{III}]$ the unknown coefficients in Williams´series, and terms of function $[b_{III}]$ are expressed in Eq. (5).

$$p_{n(r,\theta)} = \frac{2r^{n-1/2}}{G}\left\{c_n Sin\left(n-\frac{1}{2}\right)\theta\right\} \quad (5)$$

2. Results

To test the method, DIC data from the literature was used. The 3-D displacement data was obtained from three slotted thin tubes subjected to axial load and torque with R=-1, and 45° phase angle (named R-031), and 90° phase angle (named R-033). Specimens were photographed and processed with the DIC technique using 3D-VIC ® from Correlated Solutions (Columbia, SC). Figure 1 shows exemplary results from the location of displacement data taken to be analized respect ot the crack and crack tip location.

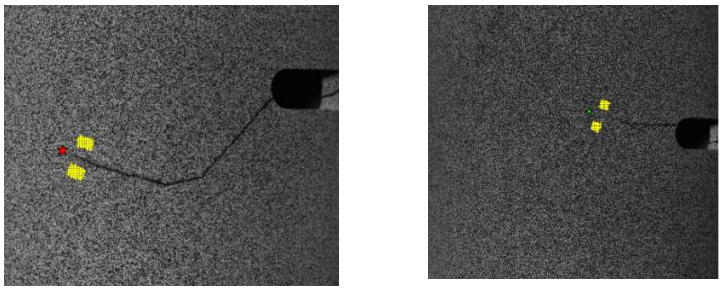

Figure 1: (a) Data source for R-031 specimen, (b) data source for R-033 specimen,

Figure 2 compares SIF obtained with proposed method with COD method for mode III for a) R-031 after 33000 loading cycles, b), R-033 after 15000 loading cycles. One can see that the proposed method reproduces the K_{III} shape of the COD behaviour through the loading cycle. For the R-033 sample, the values are very close, whereas for the sample R.031 the proposed method slightly predicts larger values. The difference may be attributed to the larger accumulated plasticity by that sample, 33000 vs 15000 cycles.

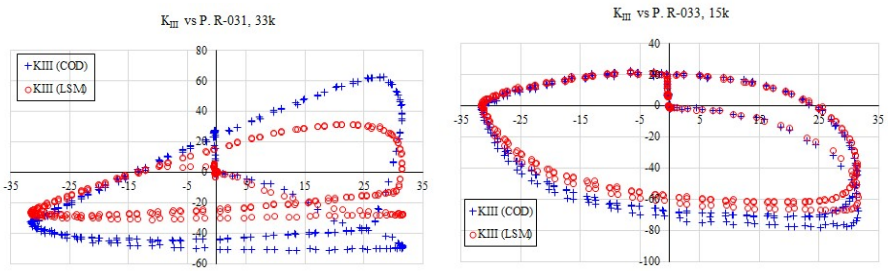

Figure 2. comparison of SIF mode III obtained with proposed method with COD measurements for a) R-031, b), R-033 and

3. Conclusion

A method to establish K_{III} from displacement data was proposed and implemented. Comparision of results with COD method show agreement for the most part.

Acknowledgements

The authors express gratitude to CAPES and to UIS VIE mobility program.