

## NEW MODEL FOR BRITTLE FRACTURE ASSESSMENT UNDER COMBINED STRESS FIELD BASED ON THE LOCAL APPROACH

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### Abstract

This study proposes the new fracture model to assess the fracture toughness under complex loading mode subjected to cracked component on the brittle fracture toughness assuming combined stress state in plastic zone near crack-tip. This model newly considers non-linear energy release rate named Local- $J$  as the elastic-plastic local fracture driving force for micro-crack nucleus in plastic zone. The effect of 3-dimensional combined stress state on local- $J$ , which is different from the effect on the linear elastic energy release rate for Griffith crack, is formulated as the Local- $J$  equivalent stress by conducting numerical analysis of unit-cell including a penny-shaped crack. Based on weakest link theory assuming this new model under combined stress field, Extended Weibull stress is derived as a new fracture parameter for cracked component. The characteristics of the proposal model is examined by predicting the critical load for pure mode II or III from fracture toughness assumed under pure mode I load. Fracture toughness assessed by this new model under mode II or III load is smaller than that assessed by conventional model. This result of numerical analysis implies the possibility of rational assessment of the effect of loading mode by applying the new model.

### 1. Introduction

Weibull stress as a new fracture parameter based on “Local Approach” has been proposed to evaluate the effect of crack-tip plastic constraint on the brittle fracture resistance. Many investigations have reported that the critical Weibull stress could be independent of specimen geometry and size, that is crack-tip plastic constraint. However, it was found that the critical Weibull stress identified under mode I load overestimated the fracture resistance under mixed mode I and II load [1]. The reason of non-applicability of conventional Weibull stress under mixed mode load was implied that the linear fracture mechanics concept, which would no longer applicable under large scale yielding, could not consider the effect of different combined stress field on the brittle fracture resistance. Authors have been presented that the local fracture driving force based on elastic-plastic fracture mechanics should be taken into account for developing a unified fracture controlling parameter independent of plastic constraint and loading modes [1]. This study proposes the new fracture model on the basis of elastic-plastic fracture driving force assuming combined stress state in plastic zone near crack-tip.

### 2. New fracture model

As the local fracture driving force for micro-crack nucleus in plastic zone, the non-linear energy release rate named Local- $J$  for a penny-shaped crack was newly considered. As schematically illustrated in Fig. 1, various combined stress states were represented by the axisymmetric stress state with equivalent Local- $J$  (the ratio of the normal stress applied to the vertical direction and transverse direction,  $\sigma_n : \sigma_t = 1 : 0.67$ ), which was named reference stress state. By conducting numerical analysis of unit cell including penny-shaped crack under parametric conditions, the normal stress applied to the vertical direction of weak spot under reference stress state, which was named as the Local- $J$  equivalent stress, was formulated by using only  $\sigma_n$  and von Mises stress  $\bar{\sigma}$  (also depending on the work-hardening exponent,  $n$ ) as shown in Eq. (1). Applying  $J$ -integral formulated under axisymmetric problem by Hutchinson et al. [2], the critical size of a micro-crack under combined stress state represented by reference stress state could be written as follows:

$$a_{cr} = \frac{J_{Local,cr}}{R(L, n) \sigma_{eq(J)} \bar{\epsilon}_{eq(J)}} ; \sigma_{eq(J)} = 2.54 [1 - \exp(-0.16n)] \left( \bar{\sigma} - \frac{\sigma_x}{\bar{\sigma}} \right) \quad (1)$$

where  $\bar{\epsilon}_{eq(J)}$  is equivalent plastic strain,  $J_{Local,cr}$  is unique fracture initiation resistance, which depends on only materials. Under the reference stress state,  $R(L, n)$  is constant. Assuming the exponential distributions of the crack size ( $p(a) = a/a^\beta$ ) and applying weakest link theory in fracture process zone subjected to stress field near crack-tip (shown in Fig. 2), a new fracture parameter  $\Sigma_{W(J)}$  was derived as shown in Eq. (2), which was newly named Extended Weibull stress. The cumulative fracture probability  $F$  evaluated with  $\Sigma_{W(J)}$  follows a two-parameter Weibull distribution which depends on only materials (shape parameter  $M=\beta-1$ ).

$$F = 1 - \exp \left[ - \left( \frac{\Sigma_{W(J),cr}}{\Sigma_u} \right)^M \right] ; \Sigma_{W(J)} = \left[ \frac{1}{V_0} \int_{\Omega} (\sigma_{eq(J)} \bar{\epsilon}_{eq(J)})^M d\Omega \right]^{\frac{1}{M}} \quad (2)$$

Here,  $V_0$  denotes unit volume and  $\Omega$  denotes micro-cracks with random-orientation and volumes of process zone for fracture near crack tip. This derivation insists that the critical Extended Weibull stress  $\Sigma_{W(J),cr}$  should be obey the unique distribution independent on loading conditions such as mixed mode or biaxial loading.

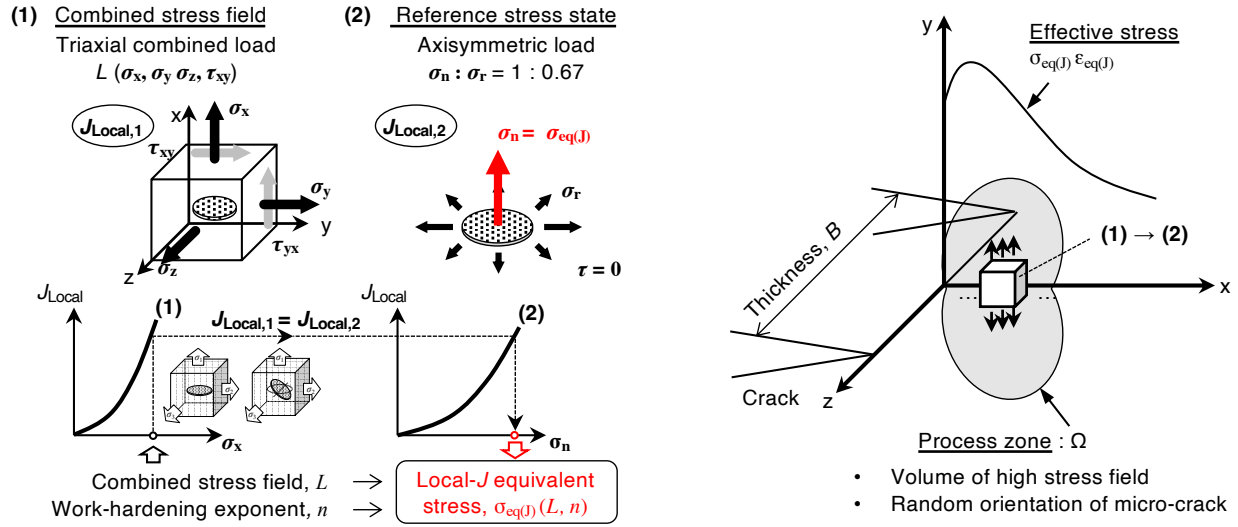


Figure 1 – Schematic illustration for definition of Local-J equivalent stress for penny shaped crack under combined stress field.

Figure 2 – Fracture process zone subjected to stress field near crack-tip.

### 3. Characteristic of fracture assessment

The characteristics of the proposal model was examined by calculating the Extended Weibull stress for cracked component assumed different conditions for loading mode. For comparison, the Weibull stress derived conventional model was calculated. In this numerical examination, Weibull stress shape parameters ( $M=5=\beta-1, m=10=2(\beta-1)$ ) assumed same materials ( $\beta=6$ ) were used for calculation. The critical load for pure mode II or III was predicted from the fracture toughness (critical Extended Weibull stress) assumed under pure mode I load. Fracture toughness assessed by this new model under mode II or III load is smaller than that assessed by using conventional model. This result of numerical analysis implies the possibility of rational assessment of the effect of loading mode effect by applying the new model.

### References

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- [2] M. Y. He and J. W. Hutchinson: The Penny-Shaped Crack and the Plane Strain Crack in an Infinite Body of Power-Law Material, *Journal of Applied Mechanics*, 48-4 (1981) 830-840.