

## ANALYTICAL SOLUTION OF CMOD COMPLIANCE FOR SINGLE EDGE NOTCHED TENSION SPECIMENS IN END-CLAMPED CONDITIONS

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### Abstract

The oil and gas industry favors to use less conservative fracture toughness measured from a single edge notched tension (SENT) specimen in the end-clamped conditions in terms of J-integral or crack-tip opening displacement (CTOD) or their resistance curves, where the elastic unloading compliance technique is usually utilized to monitor the incremental crack growth during the single specimen test. Several numerical solutions of crack mouth opening displacement (CMOD) compliance obtained from the finite element analysis (FEA) are available for the end-clamped SENT specimens. However, they have different accuracies and different applicable ranges of crack length ratio  $a/W$ , and they may be inconsistent with the existing solutions of their stress intensity factor (K) solutions for the same end-clamped SENT specimen because both the compliance and the K factor were determined separately by FEA. Based on a full-range analytical K solution, this work develops a more accurate, analytical solution of CMOD compliance equation for the end-clamped SENT specimens. Comparisons with various existing FEA results confirm the higher accuracy of the proposed analytical compliance solution. As a result, the proposed CMOD compliance solution can be used to determine more accurate crack length for the SENT testing.

### 1. Introduction

The oil and gas industry prefers to use the SENT specimens in the end-clamped conditions to measure less conservative fracture toughness or resistance curves for pipeline steels in the low-constraint conditions. The SENT toughness has been utilized in strain-based design, fitness for service and other fracture mechanics analyses for both offshore and onshore pipelines. Recently, Zhu et al. [1] delivered a comprehensive review of fracture toughness test methods with a focus on historical development efforts and recent progress using the end-clamped SENT specimens. Three primary existing fracture toughness test methods for the clamped SENT specimens are the multiple specimen test method developed by DNV for the J-integral initiation testing, the single specimen test method developed by CANMET for J-R curve and CTOD-R curve testing, and the single specimen method developed by ExxonMobil for CTOD-R curve testing. On this basis, the British Standards Institution (BSI) published the first SENT test standard BS 8571 [2] in 2014 with updates in 2018, where the elastic unloading compliance technique was adopted to monitor the incremental crack growth with use of CANMET CMOD compliance equation [3] obtained by FEA. In contrast to this FEA compliance equation, other FEA numerical compliance equations [4] have different accuracies and various applicable ranges of  $a/W$ , as shown in Fig. 1. Moreover, all numerical solutions of CMOD compliance and K factor were obtained by FEA separately. To determine a more accurate compliance solution that is consistent with the K solution, this paper develops an analytical, closed-form solution of CMOD compliance equation from a full-range K solution for the clamped SENT specimens. Comparison with available FEA results verifies the higher accuracy of the proposed CMOD compliance solution for the end-clamped SENT specimens.

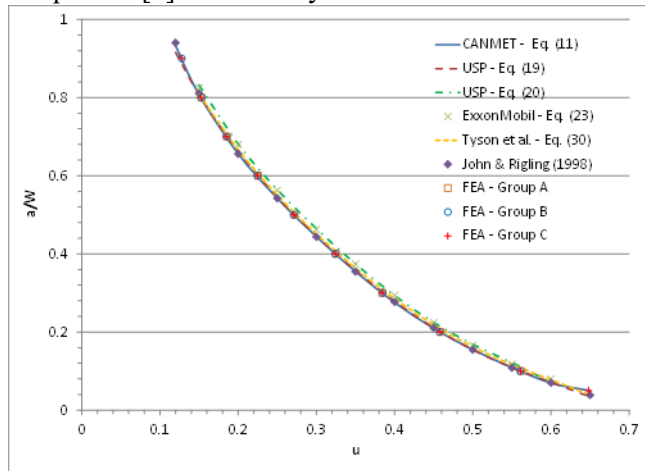


Fig. 1 - Comparison of available numerical CMOD compliance equations for clamped SENT specimens.

## 2. Results

The compliance method showed that CMOD can be determined for the end-clamped SENT specimen if one knows one geometry function of the K factor and two geometry functions of CMOD for the same single edge notched specimen under loading of pure tension and pure bending, respectively. Once CMOD is obtained, the CMOD compliance for the clamped SENT specimen is simply determined by:

$$EBC_{CMOD} = 2\alpha V\left(\frac{a}{W}\right) = 2\left(\frac{a}{W}\right)\left(V_T\left(\frac{a}{W}\right) - 6\xi_3 V_M\left(\frac{a}{W}\right)\right) \quad (1)$$

where  $V_T$  and  $V_M$  are the geometry functions of CMOD for the pure tension and bending cases,  $\xi_3$  is the geometry function of the K factor. With the solutions of  $V_T$  and  $V_M$  obtained Tada et al. (1973) and the full-range of K solution obtained Zhu [5], an analytical compliance solution can be determined for the clamped SENT specimens, as shown in Fig. 2(a). Its reserve solution is shown in Fig. 2(b) and expressed as:

$$\frac{a}{W} = 1.4747 - 6.3559u + 16.404u^2 - 30.563u^3 + 31.653u^4 - 12.79u^5 \quad (2)$$

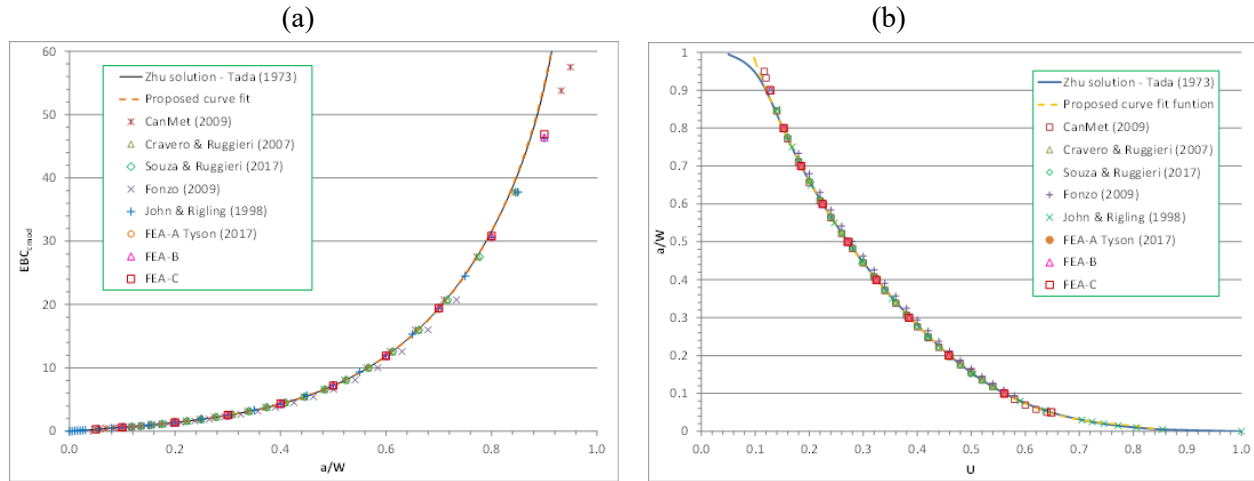


Fig. 2 – (a) Comparisons of proposed CMOD compliance solution with FEA results, (b) Comparisons of the proposed CMOD compliance reverse solution with FEA results

## 3. Conclusions

This paper developed a more accurate analytical solution of CMOD compliance and its reserve equation for end-clamped SENT specimens. The proposed compliance is consistent with the full-range K solution.

## References

- [1] Zhu XK. “Progress in Development of Fracture Toughness Test Methods for SENT Specimens,” International Journal of Pressure Vessels and Piping, Vol. 156, 2017: 40-58.
- [2] BS 8571-2014, Method of Test for Determination of Fracture Toughness in Metallic Materials using Single Edge Notched Tension (SENT) Specimens, British Standards Institution.
- [3] Shen G., Tyson WR. “Crack Size Evaluation Using Unloading Compliance in Single-Specimen Single-Edge-Notched Tension Fracture Toughness Testing,” Journal of Testing and Evaluation, Vol. 37(4), 2009, JTE102368.
- [4] Cravero S. and Ruggieri C. “Estimation Procedure of J-Resistance Curves for SE(T) Fracture Specimens Using Unloading Compliance,” Engineering Fracture Mechanics, Vol. 74, 2007: 2375-2757.
- [5] Zhu XK. “Full-Range Stress Intensity Factor Solutions for Clamped SENT Specimens,” International Journal of Pressure Vessels and Piping, Vol. 149, 2017: 1-13.

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