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THE VIRTUAL ELEMENT METHOD FOR EFFICIENT CRACK TIP LOADING ANALYSIS AND CRACK GROWTH SIMULATION

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Abstract

To precisely model crack growth, accurate calculations of crack front loading and crack deflection angles are essential. These calculations require solutions of the underlying boundary value problems (BVPs), which are typically obtained by applying numerical methods, e.g., the finite element method (FEM). However, since accuracy and computational cost of the analyses are in general competing aspects, compromises often have to be made in order to generate satisfactory results in acceptable times. In contrast, the use of more efficient methods, both for the solution of the BVP as well as for the subsequent crack tip loading analyses, can substantially lower the computational effort while maintaining desired accuracies. The virtual element method (VEM) is a fairly new discretization scheme for the numerical solution of BVPs, and can be interpreted as a generalization of the FEM. Since the VEM can handle arbitrary polytopal meshes in a straightforward manner, it provides a higher degree of flexibility in the discretization process than the FEM, which turns out to be profitable in terms of both computing times and accuracy. This holds in particular for the simulation of crack growth in 2D and 3D, sparing adaptive re-meshing or the construction of discontinuous element shape functions.

1. Point of departure

After initially being presented in 2013 by Beirão da Veiga et. al. [1], the VEM has proved to be competitive with established numerical methods in different areas of research. In the context of, e.g., engineering problems, it has been increasingly used in recent years as an alternative to the FEM. Recent applications range from linear elastic deformations to complex multiphysical problems undergoing large deformations. Thereby, the most prominent advantage of the VEM is the possibility of handling complex meshes with arbitrary polytopes (see Fig. 1) in a straightforward manner, without the explicit knowledge of shape functions inside the elements. This is obtained by replacing the sought fields by their projections onto a polynomial space, however resulting in a rank-deficient structure requiring additional stabilization.

In the context of numerical applications of fracture mechanics, the probably most attractive feature of the VEM results from the possibility to employ elements of complex shapes, which may be convex as well as concave. Consequently, crack growth simulations benefit from the fact that incremental changes in the geometry of a crack do not require any remeshing of the structure, but rather crack paths can run through already existing elements, as shown in recent works [2]. Although the method has already proved to provide an efficient tool for crack growth simulations in plane problems, there is still further research required regarding the efficient and precise evaluation of crack front loading quantities and the extension towards spatial crack problems. In particular for 3D loading analyses, the concept of configurational forces [3] proved to be very efficient for the calculation of crack front loading as well as local deflection angles. There are, however, a few numerical issues coping, e.g., with discontinuities at element edges, generating new challenges in regard to the VEM.

2. Work contents

Classical concepts of numerical fracture mechanics are adopted and implemented in connection with the VEM, carefully investigating and exploiting the advantages and opportunities the new discretization method offers in this regard. Therefore, a low order VEM is implemented for solving linear elastic mechanical BVPs and verified in connection with well-known problems of linear elastic fracture mechanics. Displacement-based methods to calculate crack front loading in terms of stress intensity factors are implemented just as the modified crack closure integral. The influence of different meshes on the accuracy of the results and the computational cost is thoroughly investigated and compared to the FEM.

Furthermore, studies on implementing advanced concepts of fracture mechanical analyses into the VEM are conducted. In this context, the research mainly focusses on the calculation of configurational forces (c. f.) and forces in material space, respectively, for two- and three-dimensional applications. The so-called configurational force method, typically used in connection with FE analyses, is employed to obtain crack driving forces from local distributions of stresses and strains in the vicinity of the crack front or tip in 2D, see Fig. 2. However, as the c. f. method is known to provide inaccurate results of the driving force, measures to improve the calculations' accuracy are provided. Finally, crack growth simulations based on the VEM are performed. Results are compared to reference solutions as well as solutions obtained by the FEM.

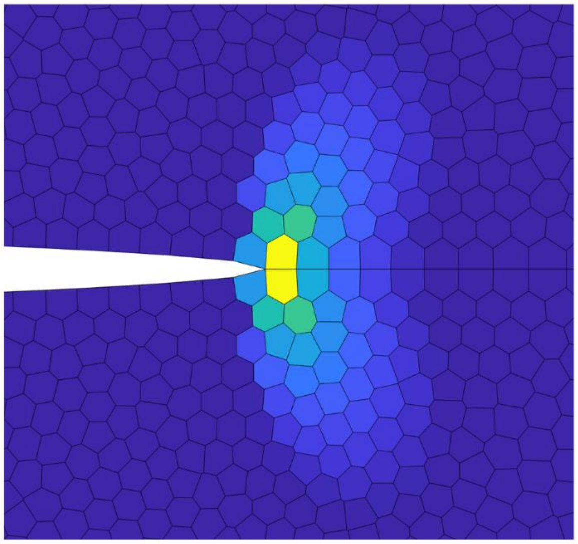

Fig.1 – Crack opening stresses on a polygonal mesh obtained by the VEM

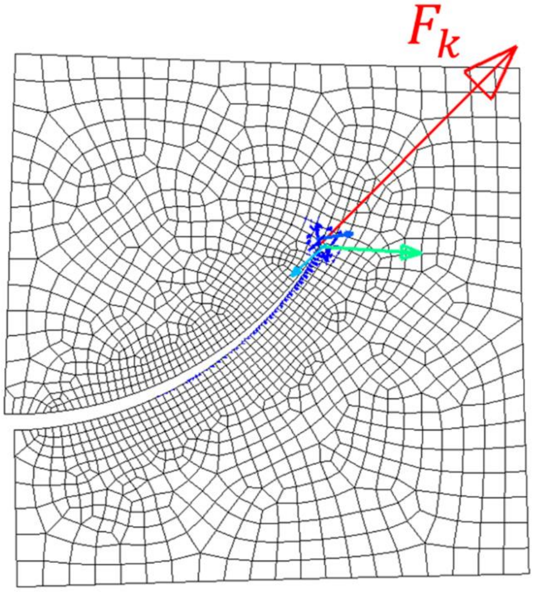

Fig.2 – Configurational forces at a crack tip obtained by the FEM

References

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