Influence of heterogeneity on failure probability based on weakest link modeling

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Abstract

Brittle failure by transgranular and intergranular mechanisms is commonly addressed by probabilistic methods based on the weakest-link concept. For homogeneous materials this approach is straightforward and well established. Different methods have been proposed in the past to incorporate the presence of heterogeneities, e.g. due to welding or segregated zones. A key issue in this context is the length that characterizes variations in the heterogeneous microstructure in relation to a representative size of the zone where brittle fracture typically has been observed to occure, i.e., fracture process zone (FPZ). Here, a new approach for weakest-link modelling of heterogeneous materials is proposed that accounts for the interplay between the different scales.

1. Introduction

For brittle materials, which typically exhibits a significant scatter in fracture toughness, weakest-link modelling is often used to describe the probability of failure, $P_f = 1 - \exp\left(-\int_V h\left(\sigma, \varepsilon_e^p\right) dV\right)$. Essential in this expression is the hazard function $h\left(\sigma, \varepsilon_e^p\right)$, which represents the probability of failure over a unit volume dV. Assuming that a heterogeneous material consists of two phases A and B, with different failure properties, the total volume V will then be the sum of subvolumes V_A and V_B. Thus, hazard functions h_A and h_B are valid in V_A and V_B, respectively, such that accumulated probability of failure becomes

$$P_f = 1 - \exp\left(-\int_{V_A} h_A\left(\sigma, \varepsilon_e^p\right) dV - \int_{V_B} h_B\left(\sigma, \varepsilon_e^p\right) \mathrm{d}V\right). \tag{1}$$

To further evaluate (1), a heterogeneity length scale l_H, characterizing the variation between phases is introduced. For small scale heterogeneity (SSH) the length scale l_H is much smaller than the size of the FPZ, here represented by the ratio between the J-integral and the yield stress, J/σ_0. In this case, when the variation of stresses and strains are small over l_H, the separation of volumes becomes insignificant as both hazard functions will be simultaneously active in a volume element $\mathrm{d}V$. By contrast, for large scale heterogeneity (LSH), the length scale l_H is much greater than the FPZ and it is possible that only one hazard function is active in the FPZ. For this case, the probability of failure depends critically on the position of the crack tip.

2. Results

The influence of the size scale effects, i.e. length l_H versus length J_0/σ_0, was investigated under small-scale-yielding (SSY) conditions using a modified boundary layer (MBL) formulation in 2D, for materials with phases having the same elastic-plastic properties. The division between phases was then done in a post-processing step by assigning the Gauss points to either phase A or B according to a predefined microstructure. Four different microstructure configurations with volume ratios $v_A = V_A/V$ and $v_B = V_B/V$ were considered as seen in Figure (1). A wide variety of materials and size scale ratios $\delta_0 = l_H/[J_0/\sigma_0]$ was investigated for full constraint (T-stress equal to zero in the MBL-formulation) and loss of constraint (negative T-stress). For a homogeneous material loaded under SSY conditions, the choice of hazard function is irrelevant. However, in the case of heterogeneous materials it matters. Thus, a few well established models were explored here. The location of the crack-tip in a microstructure was shifted within one of the repetitive domains of the configuration. Hence, for every combination of material, ratio δ_0 and constraint, the probability of failure in (1) was evaluated in the post-processing step for a large number of positions of the crack-tip in the microstructure configuration.

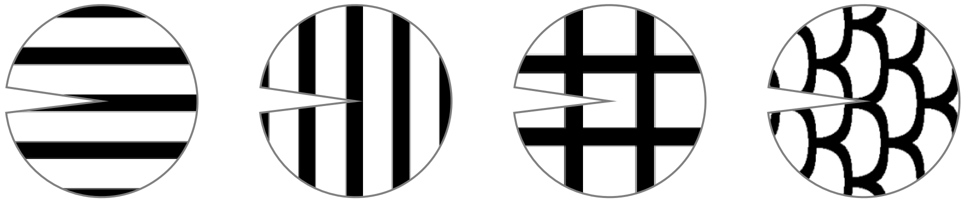

Fig. 1 – Microstructure configurations used in analysis, from left to right: horizontal lines, vertical lines, squares, scales (v_A and v_B are the same in all microstructures)

An average failure propability versus toughness curve was then calculated for each combination of parameters and microstructure. As an example, the limiting cases of SSH and LSH are shown in Fig. 2 a), and the outcome of these follow the analytical solutions for SSH and for LSH.

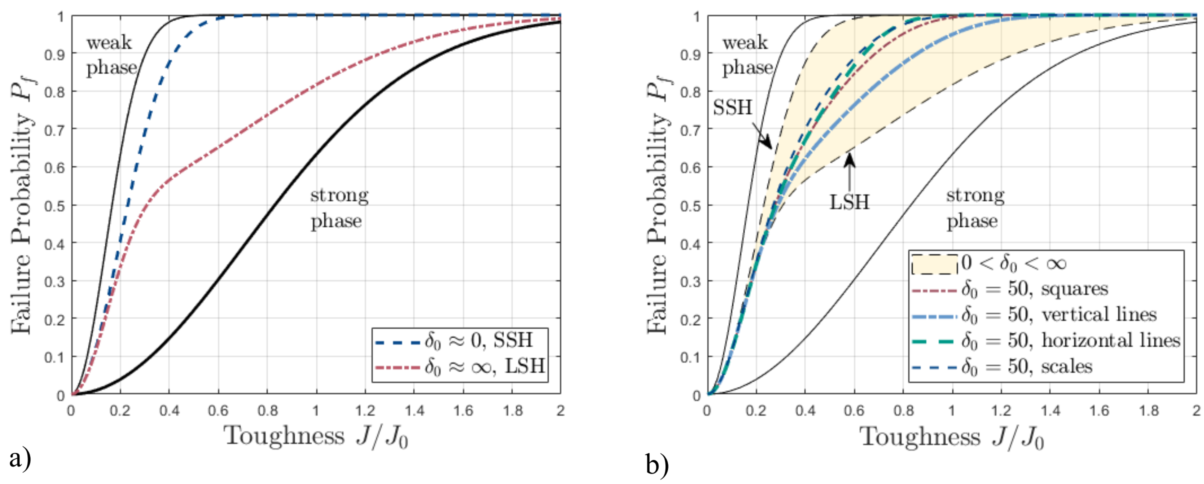

Fig. 2 – Failure probability over normalized loading for a) the limiting cases and for b) different microstructure configurations with $\delta_0 = 50$, employing the Kroon-Faleskog (2002) hazard function.

For an intermediate size scale ratio, e.g. $\delta_0 = 50$, the probability of failure curve goes from the LSH solution to the SSH solution. This is because the FPZ grows and thus successively covers a larger volume containing more microstructure units. Further, Fig. 2 b) shows that the probability of failure is significantly impacted by the different microstructure configurations.

3. Conclusions

The influence of heterogeneity on the probability of failure was investigated by analytical approaches and FE based simulations in regard to weakest-link modelling. The results show that these problems should be addressed by introducing a length scale, that characterized the variations in the heterogeneous structure.

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