PREDICTORS OF CRACK PROPAGATION

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Abstract
Stress intensity factors are viewed as specializations of a family of drivers of crack propagation, defined on three-dimensional stress fields, to two-dimensional stress fields. The question of which driver is best suited for the prediction of crack propagation in three dimensions will have to be decided on the basis of evidence developed through the application of a model development process. The procedure for rational choice of a predictor of crack propagation in metals, caused by cyclic loading, is addressed.

1. Introduction

In linear elastic fracture mechanics (LEFM) the driver of crack propagation is generally assumed to be the stress intensity factor. Specifically, we have

$$\frac{da}{dN} = C(K_{max} - K_{min})^m, \quad a_1 \le a \le a_2 \tag{1}$$

where a is crack length, N is the number of load cycles, K is the stress intensity factor C and m are experimentally determined constants, the statistical dispersion of which is not negligible. Therefore the crack length a is a random number, see for example Virkler et al. (1978). The crack lengths a_1 and a_2 define the limits of the domain of calibration.

Whereas crack propagation is a highly nonlinear, irreversible process, K is characterized by the solution of a two-dimensional problem of linear elasticity. This apparent contradiction is resolved by assuming that the nonlinear process is confined to a small volume near the crack tip, called the process zone, and the boundary conditions of the process zone are approximated by the solution of the linearly elastic problem sufficiently well for the difference to be negligible. Details are available, for example, in Szabó and Babuška (2021a). These assumptions have been validated for thin plates and through cracks over domains of calibration that include crack sizes much larger than the plate thickness. Extension of this algorithm to three dimensions and small cracks, such as corner cracks at fastener holes, is problematic for two reasons: First, the stress field in planes normal to the crack front is not the same as the two-dimensional stress field on which the stress intensity factor is defined. The two-dimensional stress field may be a reasonable approximation near the middle of the crack front, but the three-dimensional stress field is radically different in the neighborhood of points where the crack front intersects stress-free surfaces.

2. Predictors

Consider the following family of predictors:

$$P_{\alpha\lambda\varrho} = \frac{1}{\varrho^\alpha V_c}\int_{\Omega_c} |\vec{x}|^\alpha\, \sigma_1^\lambda \bar{\sigma}^{1-\lambda} dV, \quad \alpha \ge 0, \quad 0 \le \lambda \le 1, \quad \varrho > 0 \tag{2}$$

where $\vec{x}$ is the position vector in a coordinate system centered on the crack tip, σ_1 is the first principal stress, $\bar{\sigma}$ is the von Mises stress, α, λ and ϱ are adjustable parameters. The domain of integration is defined by

$$\Omega_c = \{\vec{x} \mid \sigma_1(\vec{x}) > 0, \ \vec{x}| < \varrho \} \tag{3}$$

The volume of Ω_c is denoted by V_c.

The definition of $P_{\alpha\lambda\varrho}$ is based on the idea that crack propagation is driven by a product of the principal stress and the von Mises stress averaged over a small volume, the size of which has to be determined by calibration. This assumption is purely phenomenological, it has nothing to do with the details of the highly nonlinear process involving the formation and coalescence of voids. The same assumption is used in LEFM where the stress intensity factor is the assumed driver of crack propagation.

It can be shown that in the special case when the stress field is two-dimensional and symmetric with respect to the crack (Mode I), and we let $\lambda = 1$ and $\alpha = 1/2$ then the relationship between the stress intensity factor K_I and $P_{\alpha\lambda\varrho}$ is given by

$$K_I = \frac{\pi\sqrt{2\pi}}{3} \lim_{\varrho \to 0} \left(\sqrt{\varrho} P_{\alpha\lambda\varrho}\right) \tag{4}$$

The advantage of using $P_{\alpha\lambda\varrho}$ over K_I is that $P_{\alpha\lambda\varrho}$ is defined on arbitrary stress fields whereas K_I is defined only on the two-dimensional stress field. For example, using K_I to predict the propagation of corner cracks at fastener holes introduces a model form error through violating the assumptions on which the definition of K_I is based. The smaller the corner crack, the larger the model form error will be.

Model form errors are reduced by testing alternative formulations of the predictor against experimental data, taking into account the statistical dispersion of experimental observations. A schematic view of the structure of mathematical models is shown in Fig. 1.

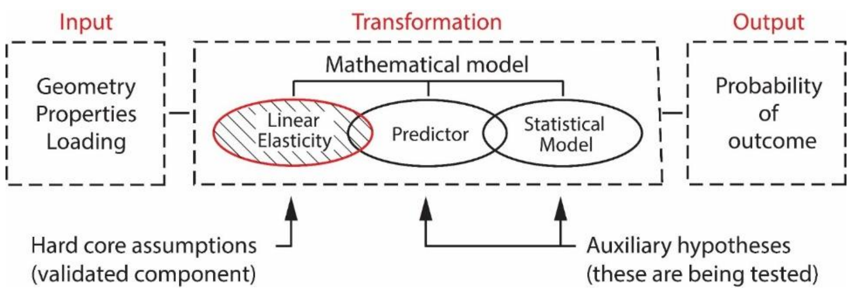

Fig.1 – The structure of mathematical models.

There are no algorithms for the formulation of predictors and statistical models. This is a creative, evolutionary and open-ended process. On the other hand, the evaluation and ranking of candidate models has to follow a strict protocol based on the rules of verification, validation and uncertainty quantification (VVUQ). For details we refer to Szabó and Babuška (2021b).

3. Conclusions

The current practice of predicting the rate of propagation of small cracks on the basis of stress intensity factors is flawed for two reasons: (a) The three-dimensional stress field at the crack front is markedly different from the two-dimensional stress field on which K_I was defined, therefore the model form error can be large, and (b) small cracks are typically outside of the domains of calibration of crack propagation models. Investigation of predictors of crack propagation, similar to the family of predictors described herein, is likely to produce significant benefits through improved reliability of crack propagation models.

References

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