

## FAST INFERENCE OF CRACK TIP POSITION AND STRESS INTENSITY FACTORS FROM DISPLACEMENT DATA

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### Abstract

Fracture prognosis and characterization efforts require knowledge of crack tip position and the configurational driving force acting on the crack. Here, we present an efficient numerical approach to determine both characteristics under a consistent theoretical framework from displacement data. The novel approach, which we refer to as the “separability approach,” utilizes the separable characteristics of the asymptotic linear elastic fracture mechanics model to expedite the search for crack tip position and is particularly useful for noisy displacement data.

### 1. Introduction

The tasks of finding the crack tip position and the configurational driving force acting on the crack are related in that the crack tip position is a necessary input when calculating the driving force. This motivates the use of observed mechanical fields (e.g., displacement, strain and stress) to define the crack tip position in cases where driving force is sought. The mechanical definition of crack tip position should be consistent with the mechanical model used for inferring the crack driving force. In the ubiquitous case of linear elastic fracture mechanics (LEFM) this equates to choosing a crack tip position that maximizes the correspondence between a given dataset and the LEFM model within an annulus surrounding the crack tip.

The proposed separability approach utilizes the multiplicative separability of the asymptotic near tip LEFM displacement field into radial and angular components to accelerate the search for crack tip position. Compared to the widely used displacement correlation approach that involves a nonlinear minimization of six parameters to identify the crack tip coordinates, the separability approach involves a single nonlinear optimization over crack tip orientation,  $K_I$ ,  $K_{II}$  and T-stress to identify the crack tip position.

### 2. Results

To assess the utility of the separability approach, we compare it to the widely used displacement correlation approach on four fronts: (1) domain of attraction, (2) robustness in application to nonideal data sets, (3) robustness relative to noise in the data, (4) computational expense. Performance is evaluated in cases involving bad starting guesses of the crack tip position, noisy data, and the case of data that does not conform to the asymptotic linear elastic fracture mechanics model, e.g., inelastic material behavior and finite geometries.

Table 1 compares the accuracy and efficiency of the separability (sep) and displacement correlation (dc) approaches in the case of non-ideal data. The number of iterations indicate the iterations of pattern search, which was incorporated to expedite computations of both approaches. Since the path taken by pattern search can vary at each call, the number of iterations and the total time taken by them for each call were averaged for 50 trials. While both approaches lead to a reasonable value of inferred crack tip location and K-field values, those computed from separability are more accurate, i.e. closer to the actual solution. Compared to the linear elastic finite element model, the crack tip shifts to the right (towards the body) for the elastic-plastic finite element case while it shifts left (away from the body) in case of a finite domain LEFM. These shifts are further exaggerated for the displacement correlation approach whereas the separability K-field solutions lie within 9% of the actual values. The total time taken to find the crack tip position using separability is also at least two orders of magnitude faster.

Table 1- A comparison of deparability (sep) and displacement correlation (dc) approaches applied to non-ideal cases: Linear Elastic Fracture Mechanics (LEFM) Finite Element (FE), Elastic Plastic Fracture Mechanics (EPFM) FE and finite domain.

	$\tilde{K}_I$	$\tilde{K}_{II}$	$\tilde{T}$	$\tilde{x}$	$\tilde{y}$	Avg. #iterations	Avg. total time taken (s)
Analytic Solution	0.80	0.00	0.00	0.00	0.00	--	--
FE Given Tip	0.72	0.01	0.00	0.00	0.00	--	--
LEFM FE- sep	0.68	-0.01	0.00	0.05	0.00	34	<b>0.01</b>
LEFM FE- dc	0.69	-0.01	0.00	0.03	0.01	134	3.31
EPFM FE- sep	0.73	0.01	-0.01	0.05	0.00	37	<b>0.07</b>
EPFM FE- dc	0.67	-0.01	-0.01	0.13	0.00	184	4.02
Finite Domain- sep	0.85	-0.02	-0.03	-0.26	0.01	48	<b>0.07</b>
Finite Domain- dc	0.90	-0.02	-0.04	-0.33	0.01	206	3.98

Not only is separability more accurate in certain cases, it also utilizes less iterations of pattern search and outperforms displacement correlation in terms of computation time. This approach to efficiently infer crack tip location and related stress intensity factors has wide-ranging applications. We exemplify its effectiveness in characterizing the crack driving force in an example case of digital image correlation as well as in inferring the crack tip location to illustrate crack initiation and propagation using molecular dynamics data (fig. 1). In both cases, the presence of noisy displacement observations and ambiguity in terms of defining crack tip (e.g., crack blunting or bridging) makes the task of extracting crack tip parameters lengthy and difficult. Within the domain of applicability of LEFM, our novel approach proves to be robust and several orders of magnitude faster than displacement correlation in computing these crack tip locations and the configurational driving force.

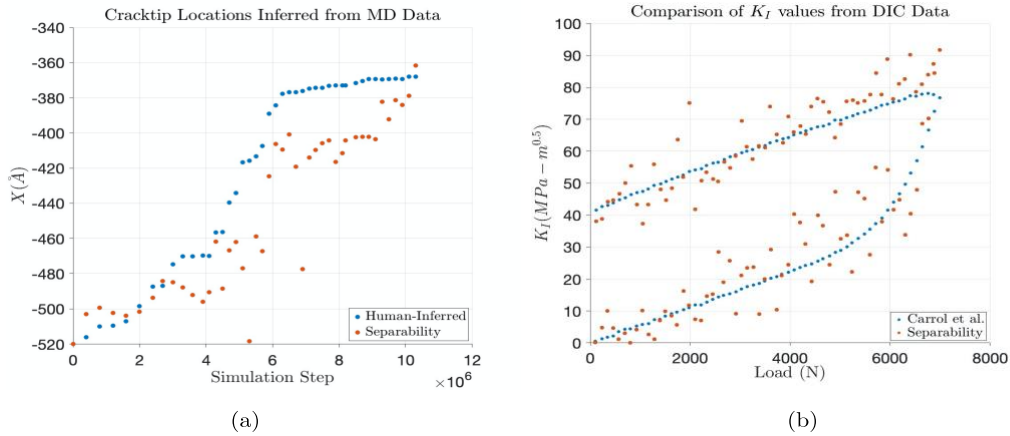


Fig. 1- Applications (a) Comparison of crack tip locations calculated by separability with the crack tips inferred from Molecular Dynamics (MD) data. (b) Comparison of  $K_I$  values from the Separability with Digital Image Correlation (DIC) data

### 3. Conclusions

Within the domain of applicability of Linear Elastic Fracture Mechanics, we demonstrate that the separability approach efficiently determines the crack tip position and the configurational driving force acting on the crack using the observed displacements. The separability approach is demonstrated to be especially effective when substantial noise exists in the observed displacement data. We envision our proposed separability method, and the associated code that will be freely available, to be useful to those doing experiments and simulations where the crack tip position is not explicitly defined, e.g., digital image correlation, finite elements with damage models and atomistic simulations of crack growth.