A Gurson-type layer model for ductile porous solids containing arbitrary ellipsoidal voids with isotropic and kinematic hardening.

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Abstract
Extensions of Gurson's model for porous ductile materials have been proposed by Madou and Leblond (2012) for general ellipsoidal cavities embedded in rigid-plastic materials, and Morin et al. (2017), for spherical voids in rigid-hardenable matrices. The aim of this work is to provide a homogenized criterion for porous ductile materials incorporating both void shape effects and isotropic and kinematic hardening. A sequential limit-analysis is performed on an ellipsoidal representative volume made of some rigid-hardenable material, containing a confocal ellipsoidal cavity. The overall plastic dissipation is obtained by using the velocity field proposed by Leblond and Gologanu (2008) which satisfies conditions of homogeneous strain rate on an arbitrary family of confocal ellipsoids. The heterogeneity of hardening is accounted for by discretizing the cell into a finite number of ellipsoids between which the quantities characterizing hardening are considered as homogeneous. The model is finally assessed through comparison of its predictions with the results of micromechanical finite element simulations. The numerical and theoretical overall yield loci are compared for various distributions of isotropic and kinematic pre-hardening with a very good agreement.

1. Introduction
Previous studies on cyclic loading have shown a significant decrease in the ductility of a material subjected to this type of loading. It was established that this decrease was related to a growth of cavities during the cycles, a phenomenon known as "ratcheting" of the porosity under cyclic loading which is the consequence of booth elasticity and hardening. In addition to the ratcheting of the porosity, void shape effects were observed under low stress triaxiality cyclic loadings (Tvergaard and Nielsen, 2010). Although there are evolutions of the Gurson model taking into account the effects of hardening, (Morin et al., 2017), or shape effects, (Madou and Leblond, 2012), there is none accounting for both phenomena. Thus, this paper presents a criterion for porous ductile materials including void shape effects as well as isotropic and kinematic hardening using sequential limit analysis, together with its validation through comparison with micromechanical finite element calculations.

2. Results
Sequential limit analysis is performed on an ellipsoidal cell containing a confocal ellipsoidal void and loaded arbitrarily through conditions of homogeneous boundary strain rate. The velocity field used to obtain the overall plastic dissipation is the one discovered by Leblond and Gologanu (2008) which satisfies conditions of homogeneous strain rate on an arbitrary family of confocal ellipsoids. Then, the material is assumed to be rigid-plastic and exhibit a mixed, isotropic and kinematic hardening. It is supposed be governed by the following criterion:

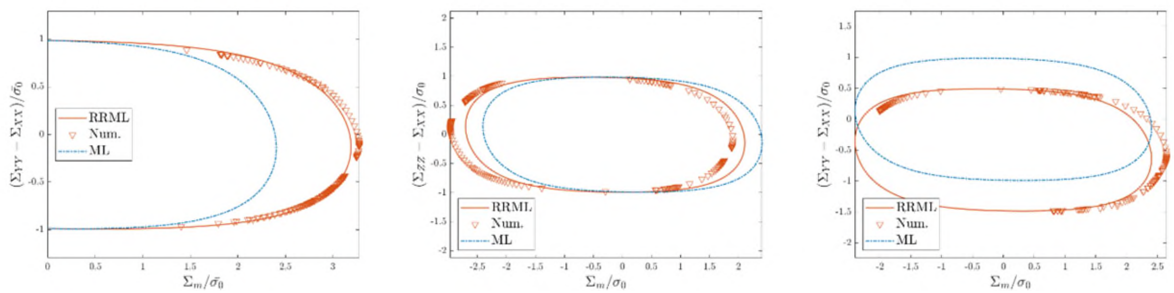
$$\phi(\boldsymbol{\sigma}) = (\boldsymbol{\sigma} - \boldsymbol{\alpha})^2_{eq} - \bar{\sigma}^2 \leq 0$$

where $\bar{\sigma}$ is the current yield stress and $\boldsymbol{\alpha}$ the traceless backstress tensor due to kinematic hardening.In order to derive the overall plastic dissipation, several approximations are introduced. The cell is discretized into a finite number of ellipsoidal layers within which the distribution of the local hardening parameters is supposed to be homogeneous, allowing the computation of the plastic dissipation.

The approximate yield criterion obtained from the sequential limit-analysis of the ellipsoidal cell reads:

$$\phi(\boldsymbol{\Sigma}) = \frac{\mathcal{Q}(\boldsymbol{\Sigma} - \boldsymbol{\Sigma}_{kine})}{\Sigma_1^2} + 2(1+g)(f+g)\cosh\left(\frac{3}{2}\frac{\mathcal{L}(\boldsymbol{\Sigma} - \boldsymbol{\Sigma}_{kine})}{\Sigma_2}\right) - (1+g)^2 - (f+g)^2 = 0$$

where $\mathcal{Q}$ and $\mathcal{L}$ are the quadratic form and the linear form defined in Madou and Leblond (2012) which depend on the orientation and semi-axes of the void. The new internal macroscopic variables Σ_1 and Σ_2 account for the effect of isotropic strain hardening and are related to the local distribution of the yield stress $\bar{\sigma}$ while $\boldsymbol{\Sigma}_{kine}$ accounts for kinematic strain hardening and is related to the local distribution of the tensor $\boldsymbol{\alpha}$. The geometrical parameters evolve in the same way as for Madou and Leblond (2012)'s model, and the local hardening parameters evolve using the velocity field of Leblond and Gologanu (2008). The model thus defined is then validated by comparing its predictions to micromechanical finite element simulations on a general ellipsoidal elementary cell subject to conditions of homogeneous boundary strain rate. Three cases are investigated by considering separately an isotropic and two kinematic hardenings. Fig.1 compares the yield surfaces obtained numerically with the theoretical model developed and the model of Madou and Leblond (2012). The proposed model is in very good agreement with the numerical results. Furthermore, the comparison with Madou Leblond (2012)'s criterion highlights the importance of considering strain hardening in the yield surfaces.



(a) (b) (c)

Fig.1 – Yield locus for a general ellipsoid $(10:5:1)$, $f = 0.01$ – Presented criterion (RRML), Numerical results (Num.), Madou Leblond (2012)'s criterion (ML). (a) Isotropic, (b) Kinematic case 1, (c) Kinematic case 2.

3. Conclusions

The applied approach provided a new macroscopic criterion for porous ductile materials incorporating the effects of isotropic and kinematic hardening and the effects of void shape. This criterion has been assessed by comparing it to finite element calculations with a good agreement.

The development of the model is completed but further numerical works will include (i) assessment of the hardening parameters' evolutions and (ii) the finite element implementation of the model.

References

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