NUMERICAL ASSESSMENT OF PHASE-FIELD APPROACH IN WESTERGAARD'S PROBLEM UNDER MIXED MODE LOADING

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Abstract
Assessment of phase-field numerical errors to classical Griffith's theory is essential to obtain feasible solutions which do not require an excessive computational cost. This work analyses the phase-field fracture approach to Westergaard's problem in terms of crack growth initiation under mode I and mixed mode I+II loading conditions.

1. Introduction
The regularized variational fracture model has shown a compelling option to other numerical fracture models due to its relevant capabilities in the simulation of complex crack trajectories [1]. However, an accurate crack growth prediction usually involves a large computational cost and finding an optimal solution becomes critical. Numerical assessment of phase field error predictions to classical Griffith's theory is commonly performed for benchmark fracture mode I problems using solutions that are already obtained by finite element analysis or through imposing boundary displacements given by just the first singular term of Williams's solution. This work analyses the phase-field fracture approach to Westergaard's problem in terms of crack growth initiation. In this way, the use of Westergaard's analytical solution, which includes all terms of the series expansion, enables a precise assessment of the phase-field model under mode I and mixed mode I+II conditions.

2. Materials and methods
The studied problem is the so-called Westergaard's crack problem which consists in an infinite plate with a crack of length $2a$, biaxially loaded with remote uniform tractions (see Fig. 1a). The expressions of the displacement field that allow for the simulation of a finite portion with dimensions w x h are presentend in [2]. Due to symmetry conditions, only half of the crack is modeled. The phase field simulations are performed using an Abaqus© user subroutine developed by Navidtehrani et al. [3]. The AT2 model is used without energy splits and the initial crack is induced by the local history variable H. The problem is subjected to two loading cases: mode I and mixed mode loading with a mixed mode ratio of 0.5. Additionally, the J-integral is calculated along a path through the exterior elements at their integration points. Two different phase field length scales l_0 are studied and four different element sizes h_e, all using structured meshes.

3. Results
As expected, and in terms of crack path, a colinear crack propagation is obtained for mode I. For mixed mode loading, the crack kinks as shown in Fig. 1b. The J-integral value calculated after crack growth for mode I loading stabilizes to a value close to the amplified effective critical energy release rate G_c* as shown in Fig. 1c. The J-integral values at crack growth initiation (see Fig.1d) converge to the critical energy release G_c for both loading cases, albeit with some differences.

4. Conclusions
In general, we show that modeling the Westergaard's problem provides valuable results in terms of error estimation in crack growth initiation for mode I and mixed mode loadings. The results converge to Griffith's theory for both loading cases considered, but with a significantly slower rate than discrete methods. In terms of crack path, it is difficult to compare the results to classical crack orientation criteria, such as the criterion of local symmetry, because the initiation of crack growth, which is intrinsic to phase field approach in opposition to discrete methods, modifies the exact imposed boundary solution of Westergard's problem. Further numerical studies will be required, although the results are still promising.

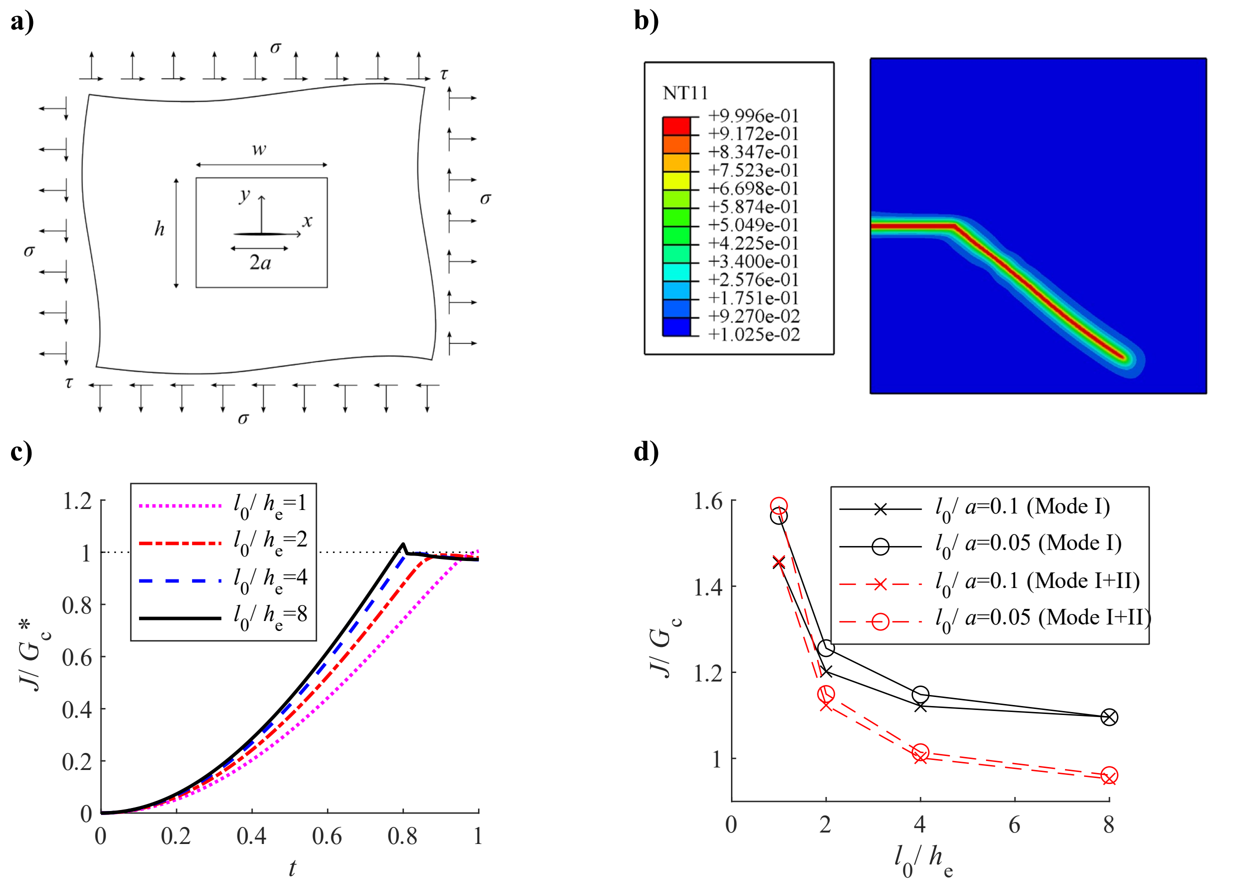

Fig.1 – (a) Westergaard's problem sketch, (b) crack path obtained under mixed mode using l_0/h_e=4 and l_0/a=0.05, (c,d) normalized J-integral values along a simulation using l_0/a=0.1 under mode I and J-integral value at crack propagation for mode I and mixed mode loading for different l_0/h_e and l_0/a values.

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