A GENERALIZED SOLUTION TO THE COMBO-CRACK PROBLEM

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Abstract

To attain accurate solutions to the crack combo problem for typical loading cases, the axisymmetric elasticity problem of the crack combo was re-visited by solving a pair of simultaneous Fredholm integral equations based on the Gauss-Lobatto quadrature. Two typical loading cases are considered, including pressure load on the surface of PSC and remote force load. The high accuracy of the obtained solutions in the whole spectra of size of the PSC and ECC was obtained and verified by finite element simulation. Our results provide generalized and more accurate solutions to the combo crack problem or equivalently the annular contact problem, which are worthy of being included in the handbook of solutions to basic crack problems.

1. Introduction

The axisymmetric elasticity problem of a crack combo containing an externally circular crack (ECC) and a coplanar concentric penny-shaped crack (PSC) has been attempted with Love's strain potential approach, which eventually comes down to solving a pair of simultaneous Fredholm integral equations. Finding the closed-form solutions to the integral equations is difficult. Approximate solutions have been proposed in power series representations (Selvadurai et al., 1987), which suffer from two major deficiencies. First, the solutions apply only to a special loading case in which uniform pressure is applied to the whole surface of the interior PSC. Secondly, the accuracy of the solution becomes unsatisfactory when the interior PSC tip is close to the ECC tip. To address these issues, we revisited this problem by considering a more general pressure loading case in which uniform pressure is applied to a circular region of any size at the center of the PSC's surface (Fig. 1a). Moreover, extension to the case of remote force load is carried out (Fig. 2a). To overcome the lower accuracy caused by power series with limited terms, we numerically solve the concerned pair of simultaneous Fredholm integral equations based on the Gauss-Lobatto quadrature. The detailed resolving process can be seen in two of our published papers (Yao et al, 2022a,b). Here, we mainly summarize the solutions to the stress intensity factors (SIFs) as follows.

2. Results

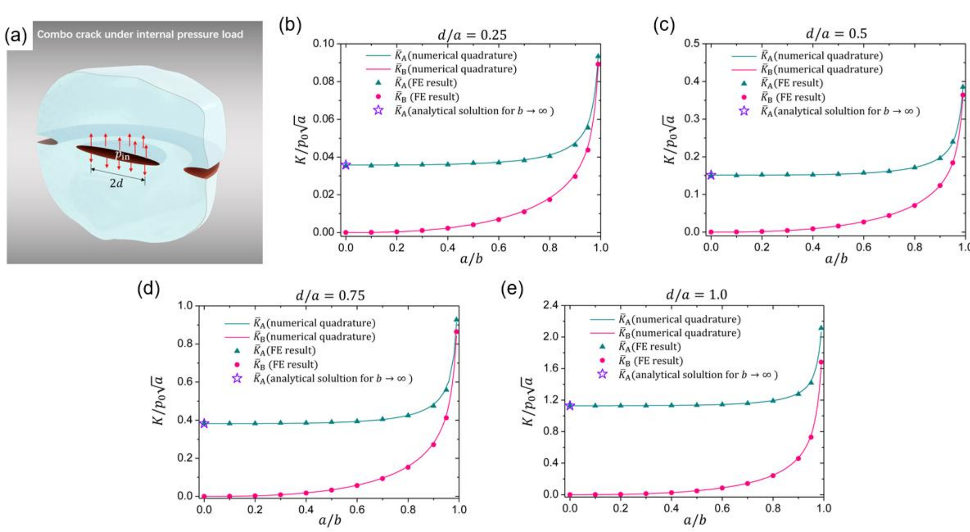

Fig.1 – (a) Schematics (section view) of a crack combo (a) under internal pressure load. The normalized SIFs at the PSC tip ($\bar{K}_A$) and the ECC tip ($\bar{K}_B$) in comparison to the FE results for cases with (b) $d/a =$

0.25, (c) $d/a = 0.5$, (d) $d/a = 0.75$, and (e) $d/a = 1.0$. The hollow star symbols represent the analytical solution for the limiting case ($b \to \infty$) in which $K/p_0\sqrt{a} = \frac{2}{\sqrt{\pi}}\left(1-\sqrt{1-d^2/a^2}\right)$.

The normalized SIFs at PSC and ECC (denoted as $\bar{K}_{\mathrm{A}}$ and $\bar{K}_{\mathrm{B}}$ respectively) we obtained are shown in Figs. 1(b-e) and Figs. 2(b,c) for both loading cases respectively in comparison with the FE results (ABAQUS, Dassault Systèmes). It can be seen that our solutions agree well with the corresponding FE solutions and the analytical solutions existing for the limiting cases.

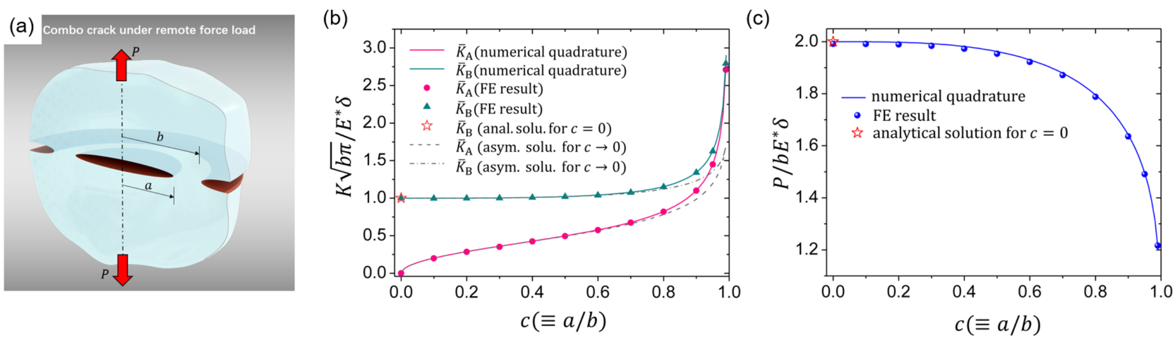

Fig.2 –(a) Schematics (section view) of a crack combo under remote force load. (b) The normalized SIFs at the PSC tip ($\bar{K}_{\mathrm{A}}$) and the ECC tip ($\bar{K}_{\mathrm{B}}$) in comparison to the FE results. The hollow star symbol represents the analytical solution $\bar{K}_{\mathrm{B}} = 1.0$ when $c \equiv a/b = 0$ (Tada et al., 2000). The asymptotic solutions $\bar{K}_{\mathrm{A}} \cong \frac{1}{\pi\sqrt{c}}\ln\frac{1+c}{1-c}$ and $\bar{K}_{\mathrm{B}} \cong \frac{4c}{\pi^2}\left[\frac{1}{2c}\ln\frac{1+c}{1-c}-1\right]+1$ for $c \equiv a/b \to 0$ are plotted with dash line and dash-dot line respectively. (c) The normalized net force as a function of c in comparison to the FE result. The hollow star symbol represents the analytical solutions $P = 2bE^*\delta$ for $c = 0$ (Tada et al., 2000).

3. Conclusions

In this paper, we developed a numerical quadrature-based technique, which enabled us to obtain more accurate numerical results of the SIFs as compared to the power series-based solutions, in the whole spectra of the sizes of the PSC and ECC. In comparison to the finite element solution, our method provides results with comparable accuracy but requires no pre-processing and post-processing and therefore is much more efficient. With the obtained solutions, we successfully predicted the critical pressure to break the annular ligament between the combo cracks. The results of this paper should be of general value to solving the related fracture and contact problems in a more precise and efficient way and are worthy of being included in the solution handbook of crack problems.

Acknowledgements

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