ON-LINE TOOL FOR ANALYSIS OF SINGULAR STRESSES AND DISPLACEMENTS IN ANISOTROPIC MULTI-MATERIAL CORNERS

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Abstract

A Python code based on a semianalytic procedure for the analysis of singular stress and displacement fields in anisotropic multi-material corners in generalized plane strain is developed. Open and closed (periodic) multi-material corners formed by isotropic, transversely isotropic or anisotropic materials with perfectly bonded interfaces or in frictionless or frictional sliding contact are considered. Many kinds of boundary conditions as free, clamped, displacement allowed or not in any given direction, and frictionless or frictional sliding contact are covered. This computational tool may help researchers who need to know the singularity exponents and the singular eigenfunctions for stresses and displacements for a corner, e.g., to improve or check their numerical results by FEM, or to verify their analytic formulas of eigenequations developed for specific stress singularity problems. With the aim of sharing this useful tool, it has been made available on research group website, where the user can introduce the corner problem parameters and the corner singularity problem is solved by a high-performance computing server.

1. Introduction

Stress singularities can take place at discontinuities in a linear elastic structure, such as non-smooth geometry, jumps in boundary/interface conditions or in material properties, usually referred to as corners. A stress singularity is a point where stresses are unbounded, and where a failure can be initiated due to high stresses. When two or more single-material wedges conform a corner, it is called multi-material corner.

In the Stroh formalism of anisotropic linear elasticity, a stress function vector is introduced as

$$\sigma_{i1} = -\varphi_{i,2}, \quad \sigma_{i2} = \varphi_{i,1}.$$

Then, in each single-material wedge the displacements and stress function vectors at the wedge tip, a singular point, can be represented by a power-law singularity

$$\mathbf{u}(r,\theta) = r^{\lambda}\{\mathbf{A}\langle\zeta_*^{\lambda}\rangle\mathbf{q} + \bar{\mathbf{A}}\langle\bar{\zeta}_*^{\lambda}\rangle\bar{\mathbf{q}}\},$$

$$\boldsymbol{\varphi}(x_1,x_2) = r^{\lambda}\{\mathbf{B}\langle\zeta_*^{\lambda}\rangle\mathbf{q} + \bar{\mathbf{B}}\langle\bar{\zeta}_*^{\lambda}\rangle\bar{\mathbf{q}}\},$$

where $\mathbf{A}, \mathbf{B}$ and $\langle\zeta_*^{\lambda}\rangle$ are complex matrices of the Stroh formalism that depend on the wedge material properties, $\langle\zeta_*^{\lambda}\rangle$ being also dependent on the wedge polar angle, $\mathbf{q}$ is an arbitrary constant vector, and λ is the corner singularity exponent. Recall that $\lambda = 0.5$ for a classical crack in homogeneous material, but it can be any complex number, with nonnegative Re λ, depending on the overall corner configuration, i.e. boundary/interface conditions, material properties and geometry of wedges. We refer to strong and weak singularities, respectively, with Re λ lower and higher than 0.5.

The computational tool presented is available on our website to compute the singularity exponent and the displacement and stress fields around the corner or along a radial edge. The user introduces the corner parameters into a form, and the results computed by a server are sent to the user automatically by email.

2. Results

This tool has been extensively and successfully tested by many numerical examples, comparing the obtained results with the results published by other authors. As an example, a frictional interface crack is studied as a bimaterial corner in the following. A composite 0/45º laminate formed by two identical laminas with $E_1 = 137.9$ GPa, $E_2 = E_3 = 14.48$ GPa, $G_{12} = G_{23} = G_{13} = 5.86$ GPa and $\nu_{12} = \nu_{23} = \nu_{13} = 0.21$ is considered. The frictional coefficient between crack faces is $\mu = 1$. The singularity exponent computed by the tool is $\lambda = 0.52254181$, and the displacements and stresses around the crack tip with *r*=1 and along a radial edge at $\theta = 180^o$ (the bonded interface) are shown in Fig. 1 and Fig. 2, respectively.

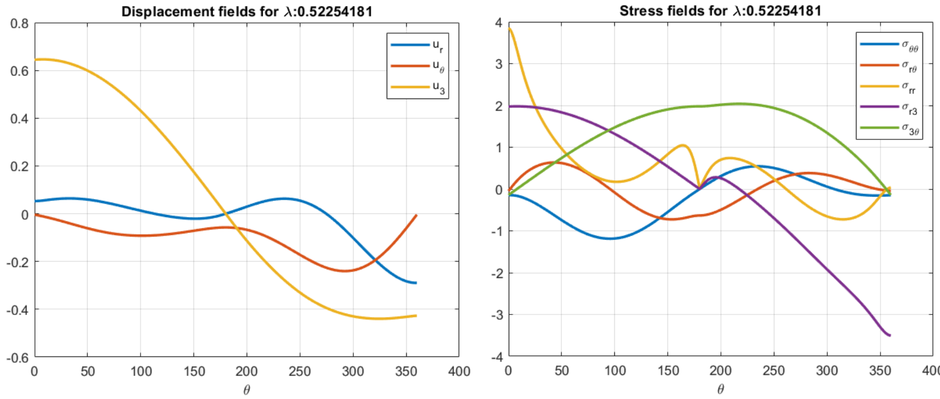

Fig.1 – Displacements and stresses around a frictional interface crack tip.

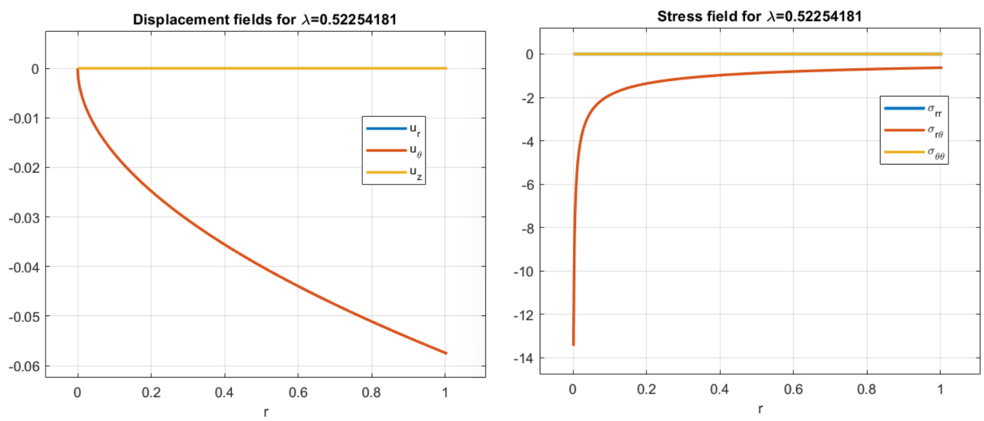

Fig.2 – Displacements and stresses along the bonded part of the interface, *r* is the distance to the crack tip.

3. Conclusions

The online tool, developed in Python language, proposed in this work allows, in an intuitive way, to obtain the singularity exponents and the stresses and displacements for anisotropic multi-material corners with general boundary and interface conditions, under generalized plane strain conditions.

The singularity exponents and singular eigenfunctions computed by this tool can be used to predict crack onset at the corner tip by the Coupled Criterion of Finite Fracture Mechanics as proposed by Yosibash et al. (2006) and García and Leguillon (2012).

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