

## 3D FRACTURE MECHANICS ANALYSIS OF THERMOMAGNETOELECTROELASTIC ANISOTROPIC SOLIDS ACCOUNTING FOR CRACK FACE CONTACT WITH FRICTION

Roman Kushnir<sup>1\*</sup>, Iaroslav Pasternak<sup>2</sup>, and Heorhiy Sulym<sup>1</sup>

<sup>1</sup>*Pidstryhach Institute for Applied Problems in Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv, Ukraine,* <sup>2</sup>*Lesya Ukrainka Volyn National University, Lutsk, Ukraine*

\* *Presenting Author email: dyrektor@iapmm.lviv.ua*

### Abstract

Thermal expansion of the material usually causes existing cracks to close, resulting in the requirement to consider contact problems. The latter are complicated since one should consider the contact of crack faces accounting for sliding, friction, and unknown contact area. This study tries to solve this task by development of the 3D boundary element approach with iterative solver, which can determine the contact zone, sliding of crack faces and account for friction between them. Moreover, multifield materials and various thermal, mechanical, electric and magnetic boundary and contact conditions can be considered.

### 1. Introduction

Thermoelectroelastic or advanced thermomagnetoelastic materials are widely used in modern technological applications. Those are smart structures (pyroelectrics, pyromagnetics, composite and natural materials containing different phases), which can convert thermal, mechanical, electric and magnetic fields. The rapid development of modern multi-field materials and micro-electro-mechanical technologies raises increasing attention to their modeling and simulation. Particular interest is focused on the issues of fracture mechanics of such materials. Since the latter are anisotropic by nature and require accounting for field interaction, their analysis is more complicated than those of thermoelastic materials.

Another challenging problem, which should be definitely accounted for, is that in the wide range of fracture mechanics problems thermal expansion of the material causes contact of crack faces, with the contact zone unknown in advance. The problem is complicated even for 2D case and 3D contact of crack faces in multifield materials is very challenging. To the best of authors' knowledge there are only a few publications considering similar issues.

### 2. Results

This study considers 3D fracture mechanics problems for thermomagnetoelastic anisotropic solids. Truly boundary integral equations are used to develop the boundary element approach for solving considered problems. These are the heat conduction dual boundary integral equations

$$\frac{1}{2} \Sigma \theta(\mathbf{x}_0) = \iint_{\partial \mathfrak{B}} \Theta^*(\mathbf{x}, \mathbf{x}_0) \Sigma h_n(\mathbf{x}) dS(\mathbf{x}) - \text{CPV} \iint_{\partial \mathfrak{B}} H^*(\mathbf{x}, \mathbf{x}_0) \Delta \theta(\mathbf{x}) dS(\mathbf{x}),$$

$$\frac{1}{2} \Delta h_n(\mathbf{x}_0) = n_j(\mathbf{x}_0) \left[ \text{CPV} \iint_{\partial \mathfrak{B}} \Theta_i^{**}(\mathbf{x}, \mathbf{x}_0) \Sigma h_n(\mathbf{x}) dS(\mathbf{x}) - \text{HFP} \iint_{\partial \mathfrak{B}} H_i^{**}(\mathbf{x}, \mathbf{x}_0) \Delta \theta(\mathbf{x}) dS(\mathbf{x}) \right],$$

and thermomagnetoelastic ones

$$\begin{aligned} \frac{1}{2} \Sigma \tilde{u}_i(\mathbf{x}_0) &= \iint_{\partial \mathfrak{B}} U_{ij}(\mathbf{x}, \mathbf{x}_0) \Sigma \tilde{t}_j(\mathbf{x}) dS(\mathbf{x}) - \text{CPV} \iint_{\partial \mathfrak{B}} T_{ij}(\mathbf{x}, \mathbf{x}_0) \Delta \tilde{u}_j(\mathbf{x}) dS(\mathbf{x}) \\ &+ \iint_{\partial \mathfrak{B}} [R_i(\mathbf{x}, \mathbf{x}_0) \Delta \theta(\mathbf{x}) + V_i(\mathbf{x}, \mathbf{x}_0) \Sigma h_n(\mathbf{x})] dS(\mathbf{x}), \end{aligned}$$

$$\frac{1}{2} \Delta \tilde{t}_l(\mathbf{x}_0) = n_j(\mathbf{x}_0) \left[ \text{CPV} \iint_{\partial \mathfrak{B}} D_{ijk}(\mathbf{x}, \mathbf{x}_0) \Sigma \tilde{t}_k(\mathbf{x}) dS(\mathbf{x}) - \text{HFP} \iint_{\partial \mathfrak{B}} S_{ijk}(\mathbf{x}, \mathbf{x}_0) \Delta \tilde{u}_k(\mathbf{x}) dS(\mathbf{x}) \right. \\ \left. + \text{CPV} \iint_{\partial \mathfrak{B}} Q_{lj}(\mathbf{x}, \mathbf{x}_0) \Delta \theta(\mathbf{x}) dS(\mathbf{x}) + \iint_{\partial \mathfrak{B}} W_{lj}(\mathbf{x}, \mathbf{x}_0) \Sigma h_n(\mathbf{x}) dS(\mathbf{x}) \right].$$

Here  $\tilde{u}_i = u_i$ ,  $\tilde{u}_4 = \phi$ ,  $\tilde{u}_5 = \psi$ ,  $u_i$  is a displacement vector;  $\phi$ ,  $\psi$  are the electric and magnetic potentials, respectively;  $\theta$  is a temperature change with respect to the reference temperature,  $h_n$  is a heat flux vector at surface with normal  $n_i$ ,  $\tilde{t}_l$  are the components of extended traction vector, which includes components of tractions, electric displacement and magnetic induction at corresponding surface;  $\Sigma f = f^+ + f^-$ ,  $\Delta f = f^+ - f^-$ , where  $f^+$  and  $f^-$  are boundary values of  $f$  at crack faces. the capital index varies from 1 to 5, while the lower case index varies from 1 to 3. Kernels of these equations are obtained previously by the authors.

The usage of these equations in contact problems is advantageous, since they relates Dirichlet (i.e.  $\theta$ ,  $\tilde{u}_l$ ) and Neumann ( $h_n$ ,  $\tilde{t}_l$ ) boundary conditions, which are natural for these tasks.

Contact conditions of crack faces are discussed in details. These are thermal, mechanical, electric and magnetic ones. In instance, when the crack faces contact each other it is assumed a perfect thermal contact, i.e.  $\Delta \theta = 0$  and  $\Sigma h_n = 0$ . In the case of crack opening at some subdomain  $S_o \subset S$  of its surface  $S$ , one should solve heat conduction boundary integral equations for  $\Delta \theta$  if thermally insulated crack ( $\Sigma h_n = 0$ ) is considered and for  $\Sigma h_n$  if perfect heat conduction of a medium, which fills in the opened crack, is assumed. The same concerns magneto-electro-mechanical contact conditions. Special attention is paid to sliding of crack faces and accounting for their friction. Iterative algorithm is developed for determination of the contact domain, direction of crack faces sliding one about another, which also accounts for friction by the Coulomb law.

Special boundary elements are introduced to account for the stress, electric displacement and magnetic induction square root singularity at the crack frontline. These boundary elements allow accurate determination of the stress, electric displacement and magnetic induction intensity factors,

$$\tilde{\mathbf{k}}^{(1)} = \lim_{\mathbf{x} \rightarrow \mathbf{x}(A)} \sqrt{\frac{\pi}{8s(\mathbf{x})}} \mathbf{L} \cdot \Delta \tilde{\mathbf{u}}^*(\mathbf{x}),$$

where  $\tilde{\mathbf{k}}^{(1)} = [K_{\text{II}}, K_{\text{I}}, K_{\text{III}}, K_D, K_B]^T$ ;  $K_{\text{I}}$ ,  $K_{\text{II}}$ ,  $K_{\text{III}}$  are the stress intensity factors;  $K_D$ ,  $K_B$  are electric displacement and magnetic induction intensity factors;  $\mathbf{L}$  is a Barnett – Lothe; and  $s(\mathbf{x})$  is an arc length evaluated from  $\mathbf{x}$  to the point  $A$  on the crack front line. Extended Sih's strain energy density fracture criterion, which accounts for mechanical, electric and magnetic fields intensity at the crack front line, is applied to estimate the behavior of a cracked multifield structural element.

### 3. Conclusions

A solid boundary element approach is developed for the analysis of cracked thermomagneto-electroelastic (pyroelectric, pyromagnetic, multiphase materials) solids, accounting for contact of crack faces. The numerical analysis held revealed that the contact of crack faces significantly influences stress, electric displacement and magnetic induction intensity at its frontline. Also, the extensive study of influence of friction on fracture parameters is provided. It is shown that mode II intensity factors are significantly affected by the account of crack faces contact and friction.