REDEFINED J-INTEGRAL AND J-INTEGRAL RANGE ΔJ AS FINITE STRAIN ELASTIC-PLASTIC CRACK PARMETERS

Hiroshi Okada[1]* and Koichiro Arai[2]

[1]*Tokyo University of Science, 2641 Yamazaki, Noda, Chiba 278-8510, Japan*
[2]*Hexagon, Kanda Square 16F, 2-2-1 Kandanishikicho, Chiyoda-ku, Tokyo 101-0054, Japan*
** Presenting Author email: hiroshi.okada@rs.tus.ac.jp*

Abstract

The summary of applications of redefined three-dimensional J-integral and J-integral range ΔJ are presented in this paper. The redefined fracture parameters were derived with a rigorous consideration on energy dissipation into a small volume in the vicinity of the crack front. It can be seen as a rigorous extension of two-dimensional T_ε^*-integral to three-dimensional problem. The equation formulations are briefly presented in this paper. Then, their applications will be presented in the conference.

1. Introduction

The J-integral (Rice [1]) has been used as the nonlinear fracture parameter for over several decades. The three-dimensional representations and domain integral methods have been presented (see [2, 3]). They have been adopted for the evaluations of J-integral. The T_ε^*-integral [4] was proposed as a general nonlinear fracture mechanics parameter but was limited in two-dimensional applications. This paper presents a summary of redefined three-dimensional J-integral that can be seen as the general extension of the T_ε^*-integral to three-dimensional problems. Its applications extend to problems with cyclic loads, nonhomogeneous materials, etc.

2. Redefined three-dimensional J-integral and J-integral range ΔJ

Arai et al. [5] redefined the three-dimensional J-integral using the domain integral representation as an extension of the T_ε^*-integral to three-dimensional and finite strain elastic-plastic problems. It measures the energy dissipating into a small volume V_ε^o per unit crack extension. Then, the ΔJ was proposed by its extension to cyclic load problem (Arai et al. [6]).

The redefined Three-dimensional J integral and J-integral range ΔJ are shown as follows.

$$\mathrm{J} = -\frac{1}{\Delta A}\int_{V^o}\left(W\delta_{1j} - \pi_{ji}\frac{\partial u_i}{\partial X_1^o}\right)\frac{\partial q}{\partial X_j^o}\mathrm{d}V^o - \frac{1}{\Delta A}\int_{V^o-V_\varepsilon^o}\left(\frac{\partial W}{\partial X_j^o} - \pi_{ji}\frac{\partial^2 u_i}{\partial X_j^o \partial X_1^o}\right)q\mathrm{d}(V^o - V_\varepsilon^o) \tag{1}$$

$$\Delta\mathrm{J} = -\frac{1}{\Delta A}\int_{V^o}\left(\Delta W\delta_{1j} - \Delta\pi_{ji}\frac{\partial \Delta u_i}{\partial X_1^o}\right)\frac{\partial q}{\partial X_j^o}\mathrm{d}V^o - \frac{1}{\Delta A}\int_{V^o-V_\varepsilon^o}\left(\frac{\partial \Delta W}{\partial X_j^o} - \Delta\pi_{ji}\frac{\partial^2 \Delta u_i}{\partial X_j^o \partial X_1^o}\right)q\mathrm{d}(V^o - V_\varepsilon^o) \tag{2}$$

Here, for simplicity of discussion, the coordinates are set so that X_1^o axis is perpendicular to the crack front and lies in the plane of the crack face. X_2^o axis is perpendicular to the plane of crack face. X_3^o lies in the tangential direction to the crack front curve. V^o is the integral domain. V_ε^o is the small volume surrounding the crack front. They are shown in Fig. 1. V^o and V_ε^o are defined in the original undeformed configuration. W, π_{ji}, u_i and X_i^o are the strain energy density, the nominal stresses, the displacements and the spatial coordinates in the original undeformed configuration. q is the scalar valued function of the spatial coordinates (X_1^o, X_2^o, X_3^o) in $V^o - V_\varepsilon^o$ and of X_3^o coordinate in V_ε^o. A typical variation of q along the crack front and the area of virtual crack propagation ΔA are shown in Fig. 1. ΔA is expressed by following integral.

$$\Delta \mathrm{A} = \int_\Delta q(X_3^o)\mathrm{d}X_3^o \tag{3}$$

The strain energy density W is given by,

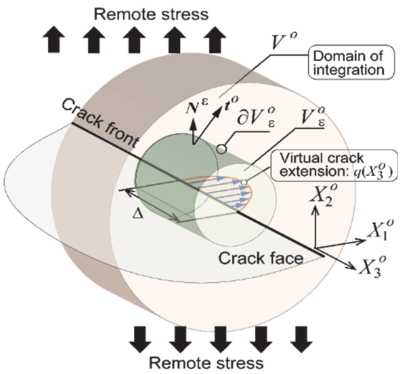

Fig. 1 - Integral domain and virtual crack extension for the J-integral and the J-integral range ΔJ.

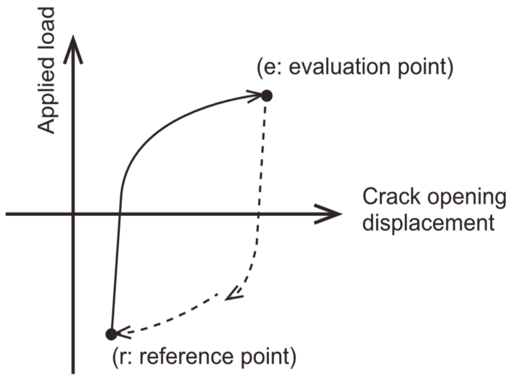

Fig. 2 - Reference and evaluation points for the evaluations of ΔW, $\Delta\pi_{ij}$ and Δu_i in the computation of the J-integral range ΔJ.

$$W = \int_0^{F_{ji}} \pi_{ij}(F_{mn})\mathrm{d}F_{ji}. \tag{4}$$

where F_{ij} are the deformation gradients ($F_{ij} = \frac{\partial x_i}{\partial X_j^o}$, x_i: coordinates of the material point in the deformed configuration).

where ΔW, $\Delta\pi_{ij}$ and Δu_i are given by,

$$\Delta W = \int_{F_{ij}|_r}^{F_{ij}|_e} \left(\pi_{ij}(F_{mn}) - \pi_{ij}\big|_r\right)\mathrm{d}F_{ji}, \quad \Delta\pi_{ij} = \pi_{ij}\big|_e - \pi_{ij}\big|_r, \ \Delta u_i = \Delta u_i|_e - \Delta u_i|_r. \tag{7}$$

Here, $(\blacksquare)|_e$ and $(\blacksquare)|_r$ represent the quatity $(\blacksquare)$ at the evaution and the reference points.

3. Summary

The authors are now seeking applications of the outcomes of present research. Because the proposed J-integral and ΔJ have the solid basis as energy dissipation and the

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